syn.1 Valuations and Satisfaction

mvl:syn:val:sec

Definition syn.1 (Valuations). Let V be a set of truth values. A *valuation* for $\mathcal{L}$ into V is a function $\mathfrak{v}$ assigning an element of V to the propositional variables of the language, i.e., $\mathfrak{v}\colon \mathrm{At}_0 \to V$.

mvl:syn:val:defn:pValue

Definition syn.2. Given a valuation $\mathfrak{v}$ into the set of truth values V of a many-valued logic $\mathbf{L}$, define the evaluation function $\overline{\mathfrak{v}}\colon \mathrm{Frm}(\mathcal{L}) \to V$ inductively by:

1. $\overline{\mathfrak{v}}(p_n) = \mathfrak{v}(p_n)$;
2. If $\star$ is a 0-place connective, then $\overline{\mathfrak{v}}(\star) = \widetilde{\star}_{\mathbf{L}}$;
3. If $\star$ is an n-place connective, then

$$\overline{\mathfrak{v}}(\star(\varphi_1, \dots, \varphi_n)) = \widetilde{\star}_{\mathbf{L}}(\overline{\mathfrak{v}}(\varphi_1), \dots, \overline{\mathfrak{v}}(\varphi_n)).$$

mvl:syn:val:defn:satisfaction

Definition syn.3 (Satisfaction). The formula φ is *satisfied* by a valuation $\mathfrak{v}$, $\mathfrak{v} \vDash_{\mathbf{L}} \varphi$, iff $\overline{\mathfrak{v}}_{\mathbf{L}}(\varphi) \in V^+$, where V^+ is the set of designated truth values of $\mathbf{L}$.

We write $\mathfrak{v} \nvDash_{\mathbf{L}} \varphi$ to mean "not $\mathfrak{v} \vDash_{\mathbf{L}} \varphi$." If Γ is a set of formulas, $\mathfrak{v} \vDash_{\mathbf{L}} \Gamma$ iff $\mathfrak{v} \vDash_{\mathbf{L}} \varphi$ for every $\varphi \in \Gamma$.

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Bibliography