## syn.1 Semantic Notions

mvl:syn:sem:sec

Suppose a many-valued logic **L** is given by a matrix. Then we can define the usual semantic notions for **L**.

**Definition syn.1.**

1. A formula $\varphi$ is *satisfiable* if for some $\mathfrak{v}$, $\mathfrak{v} \vDash \varphi$; it is *unsatisfiable* if for no $\mathfrak{v}$, $\mathfrak{v} \vDash \varphi$;
2. A formula $\varphi$ is a *tautology* if $\mathfrak{v} \vDash \varphi$ for all valuations $v$;
3. If $\Gamma$ is a set of formulas, $\Gamma \vDash \varphi$ ("$\Gamma$ entails $\varphi$") if and only if $\mathfrak{v} \vDash \varphi$ for every valuation $\mathfrak{v}$ for which $\mathfrak{v} \vDash \Gamma$.
4. If $\Gamma$ is a set of formulas, $\Gamma$ is *satisfiable* if there is a valuation $\mathfrak{v}$ for which $\mathfrak{v} \vDash \Gamma$, and $\Gamma$ is *unsatisfiable* otherwise.

We have some of the same facts for these notions as we do for the case of classical logic:

*mvl:syn:sem:prop:semanticalfacts*

**Proposition syn.2.**

1. *$\varphi$ is a tautology if and only if $\emptyset \vDash \varphi$;*
2. *If $\Gamma$ is satisfiable then every finite subset of $\Gamma$ is also satisfiable;*
3. *Monotony: if $\Gamma \subseteq \Delta$ and $\Gamma \vDash \varphi$ then also $\Delta \vDash \varphi$;* *mvl:syn:sem:def:Monotony*
4. *Transitivity: if $\Gamma \vDash \varphi$ and $\Delta \cup \{\varphi\} \vDash \psi$ then $\Gamma \cup \Delta \vDash \psi$;* *mvl:syn:sem:def:Cut*

*Proof.* Exercise. □

**Problem syn.1.** Prove Proposition syn.2

In classical logic we can connect entailment and the conditional. For instance, we have the validity of *modus ponens*: If $\Gamma \vDash \varphi$ and $\Gamma \vDash \varphi \to \psi$ then $\Gamma \vDash \psi$. Another important relationship between $\vDash$ and $\to$ in classical logic is the semantic deduction theorem: $\Gamma \vDash \varphi \to \psi$ if and only if $\Gamma \cup \{\varphi\} \vDash \psi$. These results *do not* always hold in many-valued logics. Whether they do depends on the truth function $\widetilde{\to}$.

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## Bibliography