

|              |              |                  |              |              |                |              |              |                       |              |              |
|--------------|--------------|------------------|--------------|--------------|----------------|--------------|--------------|-----------------------|--------------|--------------|
| $\simeq$     |              | $\tilde{\wedge}$ | $\mathbb{T}$ | $\mathbb{F}$ | $\tilde{\vee}$ | $\mathbb{T}$ | $\mathbb{F}$ | $\tilde{\Rightarrow}$ | $\mathbb{T}$ | $\mathbb{F}$ |
| $\mathbb{T}$ | $\mathbb{F}$ | $\mathbb{T}$     | $\mathbb{T}$ | $\mathbb{F}$ | $\mathbb{T}$   | $\mathbb{T}$ | $\mathbb{T}$ | $\mathbb{T}$          | $\mathbb{T}$ | $\mathbb{F}$ |
| $\mathbb{F}$ | $\mathbb{T}$ | $\mathbb{F}$     | $\mathbb{F}$ | $\mathbb{F}$ | $\mathbb{F}$   | $\mathbb{T}$ | $\mathbb{F}$ | $\mathbb{F}$          | $\mathbb{T}$ | $\mathbb{T}$ |

Figure 1: Truth functions for classical logic **C**.

`mvl:syn:mat:`  
`fig:tf-CL`

## syn.1 Matrices

`mvl:syn:mat:`  
`sec` A many-valued logic is defined by its language, its set of truth values  $V$ , a subset of designated truth values, and truth functions for its connective. Together, these elements are called a *matrix*.

`mvl:syn:mat:`  
`defn:matrix` **Definition syn.1 (Matrix).** A *matrix* for the logic **L** consists of:

1. a set of connectives making up a language  $\mathcal{L}$ ;
2. a set  $V \neq \emptyset$  of truth values;
3. a set  $V^+ \subseteq V$  of designated truth values;
4. for each  $n$ -place connective  $\star$  in  $\mathcal{L}$ , a truth function  $\tilde{\star} : V^n \rightarrow V$ . If  $n = 0$ , then  $\tilde{\star}$  is just an element of  $V$ .

**Example syn.2.** The matrix for classical logic **C** consists of:

1. The standard propositional language  $\mathcal{L}_0$  with  $\perp, \neg, \wedge, \vee, \rightarrow$ .
2. The set of truth values  $V = \{\mathbb{T}, \mathbb{F}\}$ .
3.  $\mathbb{T}$  is the only designated value, i.e.,  $V^+ = \{\mathbb{T}\}$ .
4. For  $\perp$ , we have  $\tilde{\perp} = \mathbb{F}$ . The other truth functions are given by the usual truth tables (see [Figure 1](#)).

## Photo Credits

## Bibliography