

## syn.1 Formulas

mvl:syn:fml:  
mvl:syn:fml:<sup>sec</sup>  
defn:formulas

**Definition syn.1 (Formula).** The set  $\text{Frm}(\mathcal{L})$  of *formulas* of a propositional language  $\mathcal{L}$  is defined inductively as follows:

1. Every **propositional variable**  $p_i$  is an atomic **formula**.
2. Every 0-place connective (propositional constant) of  $\mathcal{L}$  is an atomic **formula**.
3. If  $\star$  is an  $n$ -place connective of  $\mathcal{L}$ , and  $\varphi_1, \dots, \varphi_n$  are **formulas**, then  $\star(\varphi_1, \dots, \varphi_n)$  is a **formula**.
4. Nothing else is a **formula**.

If  $\star$  is 1-place, then  $\star(\varphi_1)$  will often be written simply as  $\star\varphi_1$ . If  $\star$  is 2-place  $\star(\varphi_1, \varphi_2)$  will often be written as  $(\varphi_1 \star \varphi_2)$ .

As usual, we will often silently leave out the outermost parentheses.

**Example syn.2.** In the standard language  $\mathcal{L}_0$ ,  $p_1 \rightarrow (p_1 \wedge \neg p_2)$  is a formula. In the language of product logic, it would be written instead as  $p_1 \rightarrow (p_1 \odot \neg p_2)$ . If we add the 1-place  $\Delta$  to the language, we would also have formulas such as  $\Delta(p_1 \wedge p_2) \rightarrow (\Delta p_1 \wedge \Delta p_2)$ .

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## Bibliography