

seq.1 Rules and Derivations

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sec For the following, let $\Gamma, \Delta, \Pi, \Lambda$ represent finite sequences of **sentences**.

Definition seq.1 (Sequent). An n -sided *sequent* is an expression of the form

$$\Gamma_1 \mid \dots \mid \Gamma_n$$

where each Γ_i is a finite (possibly empty) sequences of **sentences** of the language \mathcal{L} .

Definition seq.2 (Initial Sequent). An n -sided *initial sequent* is an n -sided sequent of the form $\varphi \mid \dots \mid \varphi$ for any **sentence** φ in the language.

If the language contains a 0-place connective \star , i.e., a propositional constant, then we also take the sequent $\dots \mid \star \mid \dots$ where \star appears in the space for the truth value associated with $\tilde{\star} \in V$, and is empty otherwise.

For each connective of an n -valued logic \mathbf{L} , there is a logical rule for each truth value that this connective can take in \mathbf{L} . **Derivations** in an n -sided sequent calculus for \mathbf{L} are trees of sequents, where the topmost sequents are initial sequents, and if a sequent stands below one or more other sequents, it must follow correctly by a rule of inference for the connectives of \mathbf{L} .

Definition seq.3 (Theorems). A **sentence** φ is a *theorem* of an n -valued logic \mathbf{L} if there is a **derivation** of the n -sequent containing φ in each position corresponding to a designated truth value of \mathbf{L} . We write $\vdash_{\mathbf{L}} \varphi$ if φ is a theorem and $\not\vdash_{\mathbf{L}} \varphi$ if it is not.

Definition seq.4 (Derivability). A **sentence** φ is *derivable from* a set of **sentences** Γ in an n -valued logic \mathbf{L} , $\Gamma \vdash_{\mathbf{L}} \varphi$, iff there is a finite subset $\Gamma_0 \subseteq \Gamma$ and a sequence Γ'_0 of the **sentences** in Γ_0 such that the following sequent has a **derivation**:

$$\Lambda_1 \mid \dots \mid \Lambda_n$$

where Λ_i is φ if position i corresponds to a designated truth value, and Γ'_0 otherwise. If φ is not **derivable** from Γ we write $\Gamma \not\vdash \varphi$.

For instance, 3-valued Łukasiewicz logic has a 3-sided sequent calculus. In a 3-sided sequent $\Gamma \mid \Pi \mid \Delta$, Γ corresponds to \mathbb{F} , Δ to \mathbb{T} , and Π to \mathbb{U} . Axioms are $\varphi \mid \varphi \mid \varphi$. Since only \mathbb{T} is designated, $\Gamma \vdash_{\mathbf{L}_3} \varphi$ iff the sequent $\Gamma \mid \Gamma \mid \varphi$ has a **derivation**. (If \mathbb{U} were also designated, we would need a **derivation** of $\Gamma \mid \varphi \mid \varphi$.)

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Bibliography