seq.1 Propositional Rules for Selected Logics

mvl:seq:prl:sec The inference rules for a connective in an n-sided sequent calculus only depend on the characteristic truth function for the connective. Thus, if some connective is defined by the same truth function in different logics, these n-sided sequent rules for the connective are the same in those logics.

Rules for ¬

The following rules for ¬ apply to Łukasiewicz and Kleene logics, and their variants.

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi}{\neg\varphi, \Gamma \mid \Pi \mid \Delta}\ \neg\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta}{\Gamma \mid \neg\varphi, \Pi \mid \Delta}\ \neg\mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta}{\Gamma \mid \Pi \mid \Delta, \neg\varphi}\ \neg\mathbb{T}$$

The following rules for ¬ apply to Gödel logic.

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta, \varphi}{\neg\varphi, \Gamma \mid \Pi \mid \Delta}\ \neg_{\mathbf{G}}\mathbb{F} \qquad \frac{\varphi, \Gamma \mid \Pi \mid \Delta}{\Gamma \mid \Pi \mid \Delta, \neg\varphi}\ \neg_{\mathbf{G}}\mathbb{T}$$

(In Gödel logic, $\neg\varphi$ can never take the value $\mathbb{U}$, so there is no rule for the middle position.)

Rules for ∧

These are the rules for ∧ in Łukasiewicz, strong Kleene, and Gödel logic.

$$\frac{\varphi, \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \land \psi, \Gamma \mid \Pi \mid \Delta}\ \land\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \Pi \mid \varphi, \Delta \qquad \Gamma \mid \psi, \Pi \mid \psi, \Delta \qquad \Gamma \mid \varphi, \psi, \Pi \mid \Delta}{\Gamma \mid \varphi \land \psi, \Pi \mid \Delta}\ \land\mathbb{U}$$

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi \qquad \Gamma \mid \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \land \psi}\ \land\mathbb{T}$$

Rules for $\lor$

These are the rules for $\lor$ in Łukasiewicz, strong Kleene, and Gödel logic.

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta \qquad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \lor \psi, \Gamma \mid \Pi \mid \Delta}\ \lor\mathbb{F}$$

$$\frac{\varphi, \Gamma \mid \varphi, \Pi \mid \Delta \qquad \psi, \Gamma \mid \psi, \Pi \mid \Delta \qquad \Gamma \mid \varphi, \psi, \Pi \mid \Delta}{\Gamma \mid \varphi \lor \psi, \Pi \mid \Delta}\ \lor\mathbb{U}$$

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \lor \psi}\ \lor\mathbb{T}$$

Rules for $\to$

These are the rules for $\to$ in Łukasiewicz logic.

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi \qquad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \to \psi, \Gamma \mid \Pi \mid \Delta}\ \to_{\mathbf{Ł_3}}\mathbb{F}$$

$$\frac{\Gamma \mid \varphi, \psi, \Pi \mid \Delta \qquad \psi, \Gamma \mid \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \to \psi, \Pi \mid \Delta}\ \to_{\mathbf{Ł_3}}\mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \psi, \Pi \mid \Delta, \psi \qquad \varphi, \Gamma \mid \varphi, \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \to \psi}\ \to_{\mathbf{Ł_3}}\mathbb{T}$$

These are the rules for $\to$ in strong Kleene logic.

$$\frac{\Gamma \mid \Pi \mid \Delta, \varphi \qquad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \to \psi, \Gamma \mid \Pi \mid \Delta}\ \to_{\mathbf{Ks}}\mathbb{F}$$

$$\frac{\psi, \Gamma \mid \psi, \Pi \mid \Delta \qquad \Gamma \mid \varphi, \psi, \Pi \mid \Delta \qquad \Gamma \mid \varphi, \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \to \psi, \Pi \mid \Delta}\ \to_{\mathbf{Ks}}\mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \to \psi}\ \to_{\mathbf{Ks}}\mathbb{T}$$

These are the rules for $\to$ in Gödel logic.

$$\frac{\Gamma \mid \varphi, \Pi \mid \Delta, \varphi \qquad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \to \psi, \Gamma \mid \Pi \mid \Delta} \to_{\mathbf{G}_3} \mathbb{F}$$

$$\frac{\Gamma \mid \psi, \Pi \mid \Delta \qquad \Gamma \mid \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \to \psi, \Pi \mid \Delta} \to_{\mathbf{G}_3} \mathbb{U}$$

$$\frac{\varphi, \Gamma \mid \psi, \Pi \mid \Delta, \psi \qquad \varphi, \Gamma \mid \varphi, \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \to \psi} \to_{\mathbf{G}_3} \mathbb{T}$$

Photo Credits

Bibliography

$$
\dfrac{
\dfrac{
\dfrac{\dfrac{\dfrac{A \mid A \mid A}{A \mid A \mid B, A}\,\mathrm{W}\mathbb{T}}{A \mid B, A \mid B, A}\,\mathrm{W}\mathbb{U}}{A \mid A, B, A \mid B, A}\,\mathrm{W}\mathbb{U}
\qquad
\dfrac{\dfrac{\dfrac{A \mid A \mid A}{A \mid A \mid A, A}\,\mathrm{W}\mathbb{T}}{A \mid A \mid B, A, A}\,\mathrm{W}\mathbb{T}}{B, A \mid A \mid B, A, A}\,\mathrm{W}\mathbb{F}
}{A \mid A \to B, A \mid B, A}\,{\to}\mathbb{U}
\qquad
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{B \mid B \mid B}{B \mid A, B \mid B}\,\mathrm{W}\mathbb{U}}{B \mid B, A \mid B}\,\mathrm{X}\mathbb{U}}{B \mid A, B, A \mid B}\,\mathrm{W}\mathbb{U}}{A, B \mid A, B, A \mid B}\,\mathrm{W}\mathbb{F}}{B, A \mid A, B, A \mid B}\,\mathrm{X}\mathbb{F}
\qquad
\dfrac{\dfrac{\dfrac{A \mid A \mid A}{A \mid A \mid B, A}\,\mathrm{W}\mathbb{T}}{B, A \mid A \mid B, A}\,\mathrm{W}\mathbb{F}}{B, B, A \mid A \mid B, A}\,\mathrm{W}\mathbb{F}
}{B, A \mid A \to B, A \mid B}\,{\to}\mathbb{U}
}{A \to B, A \mid A \to B, A \mid B}\,{\to}\mathbb{F}
$$

Figure 1: Example derivation in $\mathbf{Ł}_3$