The inference rules for a connective in an $n$-sided sequent calculus only depend on the characteristic truth function for the connective. Thus, if some connective is defined by the same truth function in different logics, these $n$-sided sequent rules for the connective are the same in those logics.

**Rules for $\neg$**

The following rules for $\neg$ apply to Lukasiewicz and Kleene logics, and their variants.

\[
\frac{\Gamma \mid \Pi \mid \Delta, \varphi}{\neg \varphi, \Gamma \mid \Pi \mid \Delta} \quad \neg F \\
\frac{\Gamma \mid \varphi, \Pi \mid \Delta}{\Gamma \mid \neg \varphi, \Pi \mid \Delta} \quad \neg U \\
\frac{\varphi, \Gamma \mid \Pi \mid \Delta}{\Gamma \mid \Pi \mid \Delta, \neg \varphi} \quad \neg T
\]

The following rules for $\neg$ apply to Gödel logic.

\[
\frac{\Gamma \mid \varphi, \Pi \mid \Delta, \varphi}{\neg \varphi, \Gamma \mid \Pi \mid \Delta} \quad \neg G F \\
\frac{\varphi, \Gamma \mid \Pi \mid \Delta}{\Gamma \mid \Pi \mid \Delta, \neg \varphi} \quad \neg G T
\]

(In Gödel logic, $\neg \varphi$ can never take the value $U$, so there is no rule for the middle position.)

**Rules for $\land$**

These are the rules for $\land$ in Lukasiewicz, strong Kleene, and Gödel logic.

\[
\frac{\varphi, \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \land \psi, \Gamma \mid \Pi \mid \Delta} \quad \land F \\
\frac{\Gamma \mid \varphi, \Pi \mid \varphi, \Delta}{\Gamma \mid \psi, \Pi \mid \psi, \Delta} \quad \land F \\
\frac{\Gamma \mid \varphi \land \psi, \Pi \mid \Delta}{\Gamma \mid \varphi, \psi, \Pi \mid \Delta} \quad \land U \\
\frac{\Gamma \mid \Pi \mid \Delta, \varphi}{\Gamma \mid \Pi \mid \Delta, \varphi \land \psi} \quad \land T \\
\frac{\Gamma \mid \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \land \psi}
\]

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Rules for $\lor$

These are the rules for $\lor$ in Łukasiewicz, strong Kleene, and Gödel logic.

\[
\begin{array}{c}
\frac{\phi, \Gamma \mid \Pi \mid \Delta \quad \psi, \Gamma \mid \Pi \mid \Delta}{\phi \lor \psi, \Gamma \mid \Pi \mid \Delta} \quad \lor_F \\
\frac{\phi, \Gamma \mid \varphi, \Pi \mid \Delta \quad \psi, \Gamma \mid \psi, \Pi \mid \Delta \quad \Gamma \mid \varphi, \psi, \Pi \mid \Delta}{\Gamma \mid \varphi \lor \psi, \Pi \mid \Delta} \quad \lor_U \\
\frac{\Gamma \mid \Pi \mid \Delta, \varphi, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \lor \psi} \quad \lor_T
\end{array}
\]

Rules for $\to$

These are the rules for $\to$ in Łukasiewicz logic.

\[
\begin{array}{c}
\frac{\Gamma \mid \Pi \mid \Delta, \varphi \quad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \to \psi, \Gamma \mid \Pi \mid \Delta} \quad \to_{L_3} F \\
\frac{\Gamma \mid \varphi, \psi, \Pi \mid \Delta \quad \psi, \Gamma \mid \psi, \Pi \mid \Delta \quad \Gamma \mid \varphi, \psi, \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \to \psi, \Pi \mid \Delta} \quad \to_{L_3} U \\
\frac{\phi, \Gamma \mid \psi, \Pi \mid \Delta, \psi \quad \varphi, \Gamma \mid \varphi, \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \to \psi} \quad \to_{L_3} T
\end{array}
\]

These are the rules for $\to$ in strong Kleene logic.

\[
\begin{array}{c}
\frac{\Gamma \mid \Pi \mid \Delta, \varphi \quad \psi, \Gamma \mid \Pi \mid \Delta}{\varphi \to \psi, \Gamma \mid \Pi \mid \Delta} \quad \to_{Ks} F \\
\frac{\psi, \Gamma \mid \psi, \Pi \mid \Delta \quad \Gamma \mid \varphi, \psi, \Pi \mid \Delta \quad \Gamma \mid \varphi, \Pi \mid \Delta, \varphi}{\Gamma \mid \varphi \to \psi, \Pi \mid \Delta} \quad \to_{Ks} U \\
\frac{\phi, \Gamma \mid \Pi \mid \Delta, \psi}{\Gamma \mid \Pi \mid \Delta, \varphi \to \psi} \quad \to_{Ks} T
\end{array}
\]

These are the rules for $\to$ in Gödel logic.
\[
\begin{align*}
\Gamma | \varphi, \Pi | \Delta, \varphi & \quad \psi, \Gamma | \Pi | \Delta \\
\varphi & \rightarrow \psi, \Gamma | \Pi | \Delta & \rightarrow_{G_3} F\\
\Gamma | \psi, \Pi | \Delta & \quad \Gamma | \Pi | \Delta, \varphi \\
\Gamma & \rightarrow \varphi, \Pi | \Delta & \rightarrow_{G_3} U\\
\varphi, \Gamma | \psi, \Pi | \Delta, \psi & \quad \varphi, \Gamma | \psi, \Pi | \Delta, \psi \\
\Gamma | \Pi | \Delta, \varphi & \rightarrow \psi & \rightarrow_{G_3} T
\end{align*}
\]

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Bibliography
Figure 1: Example derivation in $L_3$.