inf.1 Łukasiewicz logic

mvl:inf:luk:
sec

This is a short "stub" of a section on infinite-valued Łukasiewicz logic.

mvl:inf:luk:
def:lukasiewicz

Definition inf.1. Infinite-valued Łukasiewicz logic $\mathbf{Ł}_\infty$ is defined using the matrix:

1. The standard propositional language $\mathcal{L}_0$ with $\neg$, $\wedge$, $\vee$, $\rightarrow$.
2. The set of truth values V_∞.
3. 1 is the only designated value, i.e., $V^+ = \{1\}$.
4. Truth functions are given by the following functions:

$$\begin{aligned}
\widetilde{\neg}_{\mathbf{Ł}}(x) &= 1 - x \\
\widetilde{\wedge}_{\mathbf{Ł}}(x, y) &= \min(x, y) \\
\widetilde{\vee}_{\mathbf{Ł}}(x, y) &= \max(x, y) \\
\widetilde{\rightarrow}_{\mathbf{Ł}}(x, y) &= \min(1, 1 - (x - y)) = \begin{cases} 1 & \text{if } x \leq y \\ 1 - (x - y) & \text{otherwise.} \end{cases}
\end{aligned}$$

m-valued Łukasiewicz logic is defined the same, except $V = V_m$.

Proposition inf.2. *The logic $\mathbf{Ł}_3$ defined by* **??** *is the same as $\mathbf{Ł}_3$ defined by Definition inf.1.*

Proof. This can be seen by comparing the truth tables for the connectives given in **??** with the truth tables determined by the equations in Definition inf.1:

$\widetilde{\neg}$	
1	0
1/2	1/2
0	1

$\widetilde{\wedge}_{\mathbf{Ł}_3}$	1	1/2	0
1	1	1/2	0
1/2	1/2	1/2	0
0	0	0	0

$\widetilde{\vee}_{\mathbf{Ł}_3}$	1	1/2	0
1	1	1	1
1/2	1	1/2	1/2
0	1	1/2	0

$\widetilde{\rightarrow}_{\mathbf{Ł}_3}$	1	1/2	0
1	1	1/2	0
1/2	1	1	1/2
0	1	1	1

□

mvl:inf:luk:
prop:luk-infty-m

Proposition inf.3. *If $\Gamma \vDash_{\mathbf{Ł}_\infty} \psi$ then $\Gamma \vDash_{\mathbf{Ł}_m} \psi$ for all $m \geq 2$.*

Proof. Exercise. □

Problem inf.1. Prove Proposition inf.3.

In fact, the converse holds as well.

Infinite-valued Łukasiewicz logic is the most popular fuzzy logic. In the fuzzy logic literature, the conditional is often defined as $\neg\varphi \lor \psi$. The result would be an infinite-valued strong Kleene logic.

Problem inf.2. Show that $(p \to q) \lor (q \to p)$ is a tautology of $\mathbf{Ł}_\infty$.

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Bibliography