

## inf.1 Łukasiewicz logic

mvl:inf:luk:  
sec

This is a short “stub” of a section on infinite-valued Łukasiewicz logic.

mvl:inf:luk:  
def:lukasiewicz

**Definition inf.1.** Infinite-valued Łukasiewicz logic  $\mathbf{L}_\infty$  is defined using the matrix:

1. The standard propositional language  $\mathcal{L}_0$  with  $\neg, \wedge, \vee, \rightarrow$ .
2. The set of truth values  $V_\infty$ .
3. 1 is the only designated value, i.e.,  $V^+ = \{1\}$ .
4. Truth functions are given by the following functions:

$$\begin{aligned}\tilde{\neg}_{\mathbf{L}}(x) &= 1 - x \\ \tilde{\wedge}_{\mathbf{L}}(x, y) &= \min(x, y) \\ \tilde{\vee}_{\mathbf{L}}(x, y) &= \max(x, y) \\ \tilde{\rightarrow}_{\mathbf{L}}(x, y) &= \min(1, 1 - (x - y)) = \begin{cases} 1 & \text{if } x \leq y \\ 1 - (x - y) & \text{otherwise.} \end{cases}\end{aligned}$$

$m$ -valued Łukasiewicz logic is defined the same, except  $V = V_m$ .

**Proposition inf.2.** *The logic  $\mathbf{L}_3$  defined by ?? is the same as  $\mathbf{L}_3$  defined by Definition inf.1.*

*Proof.* This can be seen by comparing the truth tables for the connectives given in ?? with the truth tables determined by the equations in Definition inf.1:

$\tilde{\neg}$		$\tilde{\wedge}_{\mathbf{L}_3}$	1	1/2	0	
1	0	1	1	1/2	0	
1/2	1/2	1/2	1/2	1/2	0	
0	1	0	0	0	0	

  

$\tilde{\vee}_{\mathbf{L}_3}$	1	1/2	0	$\tilde{\rightarrow}_{\mathbf{L}_3}$	1	1/2	0	
1	1	1	1	1	1	1/2	0	□
1/2	1	1/2	1/2	1/2	1	1	1/2	
0	1	1/2	0	0	1	1	1	

mvl:inf:luk:  
prop:luk-infty-m

**Proposition inf.3.** *If  $\Gamma \vDash_{\mathbf{L}_\infty} \psi$  then  $\Gamma \vDash_{\mathbf{L}_m} \psi$  for all  $m \geq 2$ .*

*Proof.* Exercise. □

**Problem inf.1.** Prove Proposition inf.3.

In fact, the converse holds as well.

Infinite-valued Łukasiewicz logic is the most popular fuzzy logic. In the fuzzy logic literature, the conditional is often defined as  $\neg\varphi \vee \psi$ . The result would be an infinite-valued strong Kleene logic.

**Problem inf.2.** Show that  $(p \rightarrow q) \vee (q \rightarrow p)$  is a tautology of  $\mathbf{L}_\infty$ .

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## Bibliography