We may wonder if for each term there is a unique way of forming it, and there is. For each lambda term there is only one way to construct and interpret it. In the following discussion, a formation is the procedure of constructing a term using the formation rules (one or several times) of 

Lemma syn.1. A term starts with either a variable or a parenthesis.

Proof. Something counts as a term only if it is constructed according to 

Lemma syn.2. The result of an application starts with either two parentheses or a parenthesis and a variable.

Proof. If \( M \) is the result of an application, it is of the form \( (PQ) \), so it begins with a parenthesis. Since \( P \) is a term, by Lemma syn.1, it begins either with a parenthesis or a variable.

Lemma syn.3. No proper initial part of a term is itself a term.

Problem syn.1. Prove Lemma syn.3 by induction on the length of terms.

Proposition syn.4 (Unique Readability). There is a unique formation for each term. In other words, if a term \( M \) is formed by a formation, then it is the only formation that can form this term.

Proof. We prove this by induction on the formation of terms.

1. \( M \) is of the form \( x \), where \( x \) is some variable. Since the results of abstractions and applications always start with parentheses, they cannot have been used to construct \( M \); Thus, the formation of \( M \) must be a single step of 

2. \( M \) is of the form \( (\lambda x. N) \), where \( x \) is some variable and \( N \) is a term. It could not have been constructed according to 

3. \( M \) is of the form \( (PQ) \), where \( P \) and \( Q \) are terms. Since it starts with a parentheses, it cannot also be constructed by 

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A more readable paraphrase of the above proposition is as follows:

**Proposition syn.5.** A term $M$ can only be one of the following forms:

1. $x$, where $x$ is a variable uniquely determined by $M$.

2. $(\lambda x. N)$, where $x$ is a variable and $N$ is another term, both of which is uniquely determined by $M$.

3. $(PQ)$, where $P$ and $Q$ are two terms uniquely determined by $M$.

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Bibliography