

syn.1 Terms

lam:syn:trm: sec The terms of the lambda calculus are built up inductively from an infinite supply of variables v_0, v_1, \dots , the symbol “ λ ”, and parentheses. We will use x, y, z, \dots to designate variables, and M, N, P, \dots to designate terms.

lam:syn:trm: defn:term **Definition syn.1 (Terms).** The set of *terms* of the lambda calculus is defined inductively by:

- lam:syn:trm: defn:term-var 1. If x is a variable, then x is a term.
- lam:syn:trm: defn:term-abs 2. If x is a variable and M is a term, then $(\lambda x. M)$ is a term.
- lam:syn:trm: defn:term-app 3. If both M and N are terms, then (MN) is a term.

If a term $(\lambda x. M)$ is formed according to (2) we say it is the result of an *abstraction*, and the x in λx is called a *parameter*. A term (MN) formed according to (3) is the result of an *application*.

The terms defined above are fully parenthesized. This can get rather cumbersome, as the term $(\lambda x. ((\lambda x. x)(\lambda x. (xx))))$ demonstrates. We will introduce conventions for avoiding parentheses. However, the official definition makes it easy to determine how a term is constructed according to **Definition syn.1**. For example, the last step of forming the term $(\lambda x. ((\lambda x. x)(\lambda x. (xx))))$ must be abstraction where the *parameter* is x . It results by abstraction from the term $((\lambda x. x)(\lambda x. (xx)))$, which is an application of two terms. Each of these two terms is the result of an abstraction, and so on.

Problem syn.1. Describe the formation of $(\lambda g. (\lambda x. (g(xx)))(\lambda x. (g(xx))))$.

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Bibliography