int.1 The λ-Definable Functions are Closed under Composition

Lemma int.1. *The λ-definable functions are closed under composition.*

Proof. Suppose f is defined by composition from h, g_0, ..., g_{k-1}. Assuming h, g_0, ..., g_{k-1} are λ-defined by H, G_0, ..., G_{k-1}, respectively, we need to find a term F that λ-defines f. But we can simply define F by

$$F(x_0, \ldots, x_{l-1}) = H(G_0(x_0, \ldots, x_{l-1}), \ldots, G_{k-1}(x_0, \ldots, x_{l-1})).$$

In other words, the language of the lambda calculus is well suited to represent composition. $\square$

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Bibliography