int.1 The λ -Definable Functions are Closed under Composition

lam:int:com:

Lemma int.1. The λ -definable functions are closed under composition.

Proof. Suppose f is defined by composition from h, g_0, \ldots, g_{k-1} . Assuming h, g_0, \ldots, g_{k-1} are λ -defined by H, G_0, \ldots, G_{k-1} , respectively, we need to find a term F that λ -defines f. But we can simply define F by

$$F(x_0,\ldots,x_{l-1})=H(G_0(x_0,\ldots,x_{l-1}),\ldots,G_{k-1}(x_0,\ldots,x_{l-1})).$$

In other words, the language of the lambda calculus is well suited to represent composition. $\hfill\Box$

Photo Credits

Bibliography