

## int.1 The Church–Rosser Property

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sec  
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thm:church-rosser

**Theorem int.1.** *Let  $M$ ,  $N_1$ , and  $N_2$  be terms, such that  $M \rightarrow N_1$  and  $M \rightarrow N_2$ . Then there is a term  $P$  such that  $N_1 \rightarrow P$  and  $N_2 \rightarrow P$ .*

**Corollary int.2.** *Suppose  $M$  can be reduced to normal form. Then this normal form is unique.*

*Proof.* If  $M \rightarrow N_1$  and  $M \rightarrow N_2$ , by the previous theorem there is a term  $P$  such that  $N_1$  and  $N_2$  both reduce to  $P$ . If  $N_1$  and  $N_2$  are both in normal form, this can only happen if  $N_1 \equiv P \equiv N_2$ .  $\square$

Finally, we will say that two terms  $M$  and  $N$  are  $\beta$ -equivalent, or just *equivalent*, if they reduce to a common term; in other words, if there is some  $P$  such that  $M \rightarrow P$  and  $N \rightarrow P$ . This is written  $M \stackrel{\beta}{=} N$ . Using **Theorem int.1**, you can check that  $\stackrel{\beta}{=}$  is an equivalence relation, with the additional property that for every  $M$  and  $N$ , if  $M \rightarrow N$  or  $N \rightarrow M$ , then  $M \stackrel{\beta}{=} N$ . (In fact, one can show that  $\stackrel{\beta}{=}$  is the *smallest* equivalence relation having this property.)

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## Bibliography