

int.1 The Basic Primitive Recursive Functions are λ -Definable

lam:int:bas:
sec

Lemma int.1. *The functions zero, succ, and P_i^n are λ -definable.*

Proof. zero is just $\lambda x. \lambda y. y$.

The successor function succ, is defined by $\text{Succ}(u) = \lambda x. \lambda y. x(uxy)$. You should think about why this works; for each numeral \bar{n} , thought of as an iterator, and each function f , $\text{Succ}(\bar{n}, f)$ is a function that, on input y , applies f n times starting with y , and then applies it once more.

There is nothing to say about projections: $\text{Proj}_i^n(x_0, \dots, x_{n-1}) = x_i$. In other words, by our conventions, Proj_i^n is the lambda term $\lambda x_0. \dots \lambda x_{n-1}. x_i$. \square

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Bibliography