cr.1 $\beta\eta$-reduction

lam:cr:be:sec The Church–Rosser property holds for $\beta\eta$-reduction ($\xrightarrow{\beta\eta}\!\!\!\!\rightarrow$).

lam:cr:be:lem:one-par **Lemma cr.1.** *If $M \xrightarrow{\beta\eta} M'$, then $M \xRightarrow{\beta\eta} M'$.*

Proof. By induction on the derivation of $M \xrightarrow{\beta\eta} M'$. If $M \xrightarrow{\beta} M'$ by η-conversion (i.e., **??**), we use **??**. The other cases are as in **??**. □

lam:cr:be:lem:par-red **Lemma cr.2.** *If $M \xRightarrow{\beta\eta} M'$, then $M \xrightarrow{\beta\eta}\!\!\!\!\rightarrow M'$.*

Proof. Induction on the derivation of $M \xRightarrow{\beta\eta} M'$.

If the last rule is **??**, then M is $\lambda x.\, Nx$ and M' is N' for some x, N, N' where $x \notin FV(N)$ and $N \xRightarrow{\beta\eta} N'$. Thus we can first reduce $\lambda x.\, Nx$ to N by η-conversion, followed by the series of $\xrightarrow{\beta\eta}$ steps that show that $N \xrightarrow{\beta\eta}\!\!\!\!\rightarrow N'$, which holds by induction hypothesis. □

lam:cr:be:lem:str **Lemma cr.3.** *$\xrightarrow{\beta\eta}\!\!\!\!\rightarrow$ is the smallest transitive relation containing $\xRightarrow{\beta\eta}$.*

Proof. As in **??** □

lam:cr:be:thm:cr **Theorem cr.4.** *$\xrightarrow{\beta\eta}\!\!\!\!\rightarrow$ satisfies Church–Rosser property.*

Proof. By **??**, **??** and Lemma cr.3. □

Photo Credits

Bibliography