

## sc.1 The Completeness Theorem

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thm:completeness

**Theorem sc.1.** *If  $\Gamma \models \varphi$  then  $\Gamma \vdash \varphi$ .*

*Proof.* We prove the contrapositive: Suppose  $\Gamma \not\vdash \varphi$ . Then by ??, there is a prime set  $\Gamma^* \supseteq \Gamma$  such that  $\Gamma^* \not\vdash \varphi$ . Consider the canonical model  $\mathfrak{M}(\Gamma^*)$  for  $\Gamma^*$  as defined in ??. For any  $\psi \in \Gamma$ ,  $\Gamma^* \vdash \psi$ . Note that  $\Gamma^*(\Delta) = \Gamma^*$ . By the Truth Lemma (??), we have  $\mathfrak{M}(\Gamma^*), \Delta \Vdash \psi$  for all  $\psi \in \Gamma$  and  $\mathfrak{M}(\Gamma^*), \Delta \not\vdash \varphi$ . This shows that  $\Gamma \not\models \varphi$ .  $\square$

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## Bibliography