sc.1 The Canonical Model

int:sc:mod: sec The worlds in our model will be finite sequences σ of natural numbers, i.e., $\sigma \in \mathbb{N}^*$. Note that $\mathbb{N}^*$ is inductively defined by:

1. $\Lambda \in \mathbb{N}^*$.
2. If $\sigma \in \mathbb{N}^*$ and $n \in \mathbb{N}$, then $\sigma.n \in \mathbb{N}^*$ (where $\sigma.n$ is $\sigma \frown \langle n \rangle$ and $\sigma \frown \sigma'$ is the concatenation if σ and σ').
3. Nothing else is in $\mathbb{N}^*$.

So we can use $\mathbb{N}^*$ to give inductive definitions.

Let $\langle \psi_1, \chi_1 \rangle$, $\langle \psi_2, \chi_s \rangle$, ..., be an enumeration of all pairs of formulas. Given a set of formulas Δ, define $\Delta(\sigma)$ by induction as follows:

1. $\Delta(\Lambda) = \Delta$
2. $\Delta(\sigma.n) =$

$$\begin{cases} (\Delta(\sigma) \cup \{\psi_n\})^* & \text{if } \Delta(\sigma) \cup \{\psi_n\} \nvdash \chi_n \\ \Delta(\sigma) & \text{otherwise} \end{cases}$$

Here by $(\Delta(\sigma) \cup \{\psi_n\})^*$ we mean the prime set of formulas which exists by **??** applied to the set $\Delta(\sigma) \cup \{\psi_n\}$ and the formula χ_n. Note that by this definition, if $\Delta(\sigma) \cup \{\psi_n\} \nvdash \chi_n$, then $\Delta(\sigma.n) \vdash \psi_n$ and $\Delta(\sigma.n) \nvdash \chi_n$. Note also that $\Delta(\sigma) \subseteq \Delta(\sigma.n)$ for any n. If Δ is prime, then $\Delta(\sigma)$ is prime for all σ.

int:sc:mod: defn:canonical-model **Definition sc.1.** Suppose Δ is prime. Then the *canonical model* $\mathfrak{M}(\Delta)$ for Δ is defined by:

1. $W = \mathbb{N}^*$, the set of finite sequences of natural numbers.
2. R is the partial order according to which $R\sigma\sigma'$ iff σ is an initial segment of σ' (i.e., $\sigma' = \sigma \frown \sigma''$ for some sequence σ'').
3. $V(p) = \{\sigma : p \in \Delta(\sigma)\}$.

It is easy to verify that R is indeed a partial order. Also, the monotonicity condition on V is satisfied. Since $\Delta(\sigma) \subseteq \Delta(\sigma.n)$ we get $\Delta(\sigma) \subseteq \Delta(\sigma')$ whenever $R\sigma\sigma'$ by induction on σ.

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Bibliography