

sem.1 Semantic Notions

int:sem:sem:
sec

Definition sem.1. We say φ is *true in the model* $\mathfrak{M} = \langle W, R, V \rangle$, $\mathfrak{M} \Vdash \varphi$, iff $\mathfrak{M}, w \Vdash \varphi$ for all $w \in W$. φ is *valid*, $\vDash \varphi$, iff it is true in all models. We say a set of **formulas** Γ *entails* φ , $\Gamma \vDash \varphi$, iff for every model \mathfrak{M} and every w such that $\mathfrak{M}, w \Vdash \Gamma$, $\mathfrak{M}, w \Vdash \varphi$.

Proposition sem.2.

int:sem:sem:
prop:sat-entails
int:sem:sem:
prop:sat-entails1
int:sem:sem:
prop:sat-entails2

1. If $\mathfrak{M}, w \Vdash \Gamma$ and $\Gamma \vDash \varphi$, then $\mathfrak{M}, w \Vdash \varphi$.
2. If $\mathfrak{M} \Vdash \Gamma$ and $\Gamma \vDash \varphi$, then $\mathfrak{M} \Vdash \varphi$.

Proof. 1. Suppose $\mathfrak{M} \Vdash \Gamma$. Since $\Gamma \vDash \varphi$, we know that if $\mathfrak{M}, w \Vdash \Gamma$, then $\mathfrak{M}, w \Vdash \varphi$. Since $\mathfrak{M}, u \Vdash \Gamma$ for all every $u \in W$, $\mathfrak{M}, w \Vdash \Gamma$. Hence $\mathfrak{M}, w \Vdash \varphi$.

2. Follows immediately from (1). □

int:sem:sem:
defn:restrict

Definition sem.3. Suppose \mathfrak{M} is a relational model and $w \in W$. The *restriction* $\mathfrak{M}_w = \langle W_w, R_w, V_w \rangle$ of \mathfrak{M} to w is given by:

$$\begin{aligned} W_w &= \{u \in W : Rwu\}, \\ R_w &= R \cap (W_w)^2, \text{ and} \\ V_w(p) &= V(p) \cap W_w. \end{aligned}$$

int:sem:sem:
prop:restrict

Proposition sem.4. $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}_w \Vdash \varphi$.

Problem sem.1. Prove **Proposition sem.4**.

Proposition sem.5. Suppose for every model \mathfrak{M} such that $\mathfrak{M} \Vdash \Gamma$, $\mathfrak{M} \Vdash \varphi$. Then $\Gamma \vDash \varphi$.

Proof. Suppose that $\mathfrak{M}, w \Vdash \Gamma$. By the **Proposition sem.4** applied to every $\psi \in \Gamma$, we have $\mathfrak{M}_w \Vdash \Gamma$. By the assumption, we have $\mathfrak{M}_w \Vdash \varphi$. By **Proposition sem.4** again, we get $\mathfrak{M}, w \Vdash \varphi$. □

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Bibliography