

int.1 Axiomatic Derivations

int:int:axd: Axiomatic **derivations** for intuitionistic propositional logic are the conceptually
 sec simplest, and historically first, **derivation** systems. They work just as in classical propositional logic.

Definition int.1 (Derivability). If Γ is a set of **formulas** of \mathcal{L} then a **derivation** from Γ is a finite sequence $\varphi_1, \dots, \varphi_n$ of **formulas** where for each $i \leq n$ one of the following holds:

1. $\varphi_i \in \Gamma$; or
2. φ_i is an axiom; or
3. φ_i follows from some φ_j and φ_k with $j < i$ and $k < i$ by modus ponens, i.e., $\varphi_k \equiv \varphi_j \rightarrow \varphi_i$.

Definition int.2 (Axioms). The set of Ax_0 of *axioms* for the intuitionistic propositional logic are all **formulas** of the following forms:

int:int:axd:	$(\varphi \wedge \psi) \rightarrow \varphi$	(1)
ax:land1	$(\varphi \wedge \psi) \rightarrow \psi$	(2)
int:int:axd:	$\varphi \rightarrow (\psi \rightarrow (\varphi \wedge \psi))$	(3)
ax:land2	$\varphi \rightarrow (\varphi \vee \psi)$	(4)
int:int:axd:	$\varphi \rightarrow (\psi \vee \varphi)$	(5)
ax:lor1	$(\varphi \rightarrow \chi) \rightarrow ((\psi \rightarrow \chi) \rightarrow ((\varphi \vee \psi) \rightarrow \chi))$	(6)
int:int:axd:	$\varphi \rightarrow (\psi \rightarrow \varphi)$	(7)
ax:lor2	$(\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \chi))$	(8)
int:int:axd:	$\perp \rightarrow \varphi$	(9)
ax:lor3		
int:int:axd:		
ax:lif1		
int:int:axd:		
ax:lif2		
int:int:axd:		
ax:lfalse1		

Definition int.3 (Derivability). A formula φ is **derivable** from Γ , written $\Gamma \vdash \varphi$, if there is a **derivation** from Γ ending in φ .

Definition int.4 (Theorems). A formula φ is a **theorem** if there is a **derivation** of φ from the empty set. We write $\vdash \varphi$ if φ is a theorem and $\not\vdash \varphi$ if it is not.

Proposition int.5. *If $\Gamma \vdash \varphi$ in intuitionistic logic, $\Gamma \vdash \varphi$ in classical logic. In particular, if φ is an intuitionistic theorem, it is also a classical theorem.*

Proof. Every intuitionistic axiom is also a classical axiom, so every **derivation** in intuitionistic logic is also a **derivation** in classical logic. \square

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Bibliography