

## art.1 Coding Formulas

inc:art:frm:  
sec

**Proposition art.1.** *The relation  $\text{Atom}(x)$  which holds iff  $x$  is the Gödel number of an atomic formula, is primitive recursive.*

*Proof.* The number  $x$  is the Gödel number of an atomic formula iff one of the following holds:

1. There are  $n, j < x$ , and  $z < x$  such that for each  $i < n$ ,  $\text{Term}((z)_i)$  and  $x =$

$$\#P_j^n(\# \frown \text{flatten}(z) \frown \#)\#.$$

2. There are  $z_1, z_2 < x$  such that  $\text{Term}(z_1)$ ,  $\text{Term}(z_2)$ , and  $x =$

$$\#=(\# \frown z_1 \frown \#, \# \frown z_2 \frown \#)\#.$$

3.  $x = \# \perp \#$ .

4.  $x = \# \top \#$ . □

inc:art:frm:  
prop:frm-primrec

**Proposition art.2.** *The relation  $\text{Frm}(x)$  which holds iff  $x$  is the Gödel number of a formula is primitive recursive.*

*Proof.* A sequence of symbols  $s$  is a formula iff there is formation sequence  $s_0, \dots, s_{k-1} = s$  of formula which records how  $s$  was formed from atomic formulas according to the formation rules. The code for each  $s_i$  (and indeed of the code of the sequence  $\langle s_0, \dots, s_{k-1} \rangle$ ) is less than the code  $x$  of  $s$ . □

**Problem art.1.** Give a detailed proof of [Proposition art.2](#) along the lines of the first proof of ??.

inc:art:frm:  
prop:freeocc-primrec

**Proposition art.3.** *The relation  $\text{FreeOcc}(x, z, i)$ , which holds iff the  $i$ -th symbol of the formula with Gödel number  $x$  is a free occurrence of the variable with Gödel number  $z$ , is primitive recursive.*

*Proof.* Exercise. □

**Problem art.2.** Prove [Proposition art.3](#). You may make use of the fact that any substring of a formula which is a formula is a sub-formula of it.

**Proposition art.4.** *The property  $\text{Sent}(x)$  which holds iff  $x$  is the Gödel number of a sentence is primitive recursive.*

*Proof.* A sentence is a formula without free occurrences of variables. So  $\text{Sent}(x)$  holds iff

$$(\forall i < \text{len}(x)) (\forall z < x)$$

$$((\exists j < z) z = \#v_j\# \rightarrow \neg \text{FreeOcc}(x, z, i)). \quad \square$$

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**Bibliography**