

tab.1 Quantifier Rules

fol:tab:qrl: Rules for \forall sec

$$\frac{\mathbb{T} \forall x \varphi(x)}{\mathbb{T} \varphi(t)} \forall \mathbb{T} \qquad \frac{\mathbb{F} \forall x \varphi(x)}{\mathbb{F} \varphi(a)} \forall \mathbb{F}$$

In $\forall \mathbb{T}$, t is a closed term (i.e., one without variables). In $\forall \mathbb{F}$, a is a **constant symbol** which must not occur anywhere in the branch above $\forall \mathbb{F}$ rule. We call a the *eigenvariable* of the $\forall \mathbb{F}$ inference.¹

Rules for \exists

$$\frac{\mathbb{T} \exists x \varphi(x)}{\mathbb{T} \varphi(a)} \exists \mathbb{T} \qquad \frac{\mathbb{F} \exists x \varphi(x)}{\mathbb{F} \varphi(t)} \exists \mathbb{F}$$

Again, t is a closed term, and a is a **constant symbol** which does not occur in the branch above the $\exists \mathbb{T}$ rule. We call a the *eigenvariable* of the $\exists \mathbb{T}$ inference.

The condition that an eigenvariable not occur in the branch above the $\forall \mathbb{F}$ or $\exists \mathbb{T}$ inference is called the *eigenvariable condition*.

Recall the convention that when φ is a **formula** with the **variable** x free, we indicate this by writing $\varphi(x)$. In the same context, $\varphi(t)$ then is short for $\varphi[t/x]$. So we could also write the $\exists \mathbb{F}$ rule as: explanation

$$\frac{\mathbb{F} \exists x \varphi}{\mathbb{F} \varphi[t/x]} \exists \mathbb{F}$$

Note that t may already occur in φ , e.g., φ might be $P(t, x)$. Thus, inferring $\mathbb{F} P(t, t)$ from $\mathbb{F} \exists x P(t, x)$ is a correct application of $\exists \mathbb{F}$. However, the eigenvariable conditions in $\forall \mathbb{F}$ and $\exists \mathbb{T}$ require that the **constant symbol** a does not occur in φ . So, you cannot correctly infer $\mathbb{F} P(a, a)$ from $\mathbb{F} \forall x P(a, x)$ using $\forall \mathbb{F}$.

In $\forall \mathbb{T}$ and $\exists \mathbb{F}$ there are no restrictions on the term t . On the other hand, in the $\exists \mathbb{T}$ and $\forall \mathbb{F}$ rules, the eigenvariable condition requires that the **constant symbol** a does not occur anywhere in the branches above the respective inference. It is necessary to ensure that the system is sound. Without this condition, the following would be a closed **tableau** for $\exists x \varphi(x) \rightarrow \forall x \varphi(x)$: explanation

¹We use the term “eigenvariable” even though a in the above rule is a **constant symbol**. This has historical reasons.

1.	$\mathbb{F} \exists x \varphi(x) \rightarrow \forall x \varphi(x)$	Assumption
2.	$\mathbb{T} \exists x \varphi(x)$	$\rightarrow \mathbb{F} 1$
3.	$\mathbb{F} \forall x \varphi(x)$	$\rightarrow \mathbb{F} 1$
4.	$\mathbb{T} \varphi(a)$	$\exists \mathbb{T} 2$
5.	$\mathbb{F} \varphi(a)$	$\forall \mathbb{F} 3$
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However, $\exists x \varphi(x) \rightarrow \forall x \varphi(x)$ is not valid.

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Bibliography