

tab.1 Quantifier Rules

fol:tab:qrl:
sec **Rules for \forall**

$$\frac{\mathbb{T} \forall x \varphi(x)}{\mathbb{T} \varphi(t)} \forall \mathbb{T} \qquad \frac{\mathbb{F} \forall x \varphi(x)}{\mathbb{F} \varphi(a)} \forall \mathbb{F}$$

In $\forall \mathbb{T}$, t is a closed term (i.e., one without variables). In $\forall \mathbb{F}$, a is a **constant symbol** which must not occur anywhere in the branch above $\forall \mathbb{F}$ rule. We call a the *eigenvariable* of the $\forall \mathbb{F}$ inference.

Rules for \exists

$$\frac{\mathbb{T} \exists x \varphi(x)}{\mathbb{T} \varphi(a)} \exists \mathbb{T} \qquad \frac{\mathbb{F} \exists x \varphi(x)}{\mathbb{F} \varphi(t)} \exists \mathbb{F}$$

Again, t is a closed term, and a is a **constant symbol** which does not occur in the branch above the $\exists \mathbb{F}$ rule. We call a the *eigenvariable* of the $\exists \mathbb{F}$ inference.

The condition that an eigenvariable not occur in the branch above the $\forall \mathbb{F}$ or $\exists \mathbb{T}$ inference is called the *eigenvariable condition*.

We use the term “eigenvariable” even though a in the above rules is a **constant symbol**. This has historical reasons. explanation

In $\forall \mathbb{T}$ and $\exists \mathbb{F}$ there are no restrictions on the term t . On the other hand, in the $\exists \mathbb{T}$ and $\forall \mathbb{F}$ rules, the eigenvariable condition requires that the **constant symbol** a does not occur anywhere in the branches above the respective inference. It is necessary to ensure that the system is sound. Without this condition, the following would be a closed **tableau** for $\exists x \varphi(x) \rightarrow \forall x \varphi(x)$:

1.	$\mathbb{F} \exists x \varphi(x) \rightarrow \forall x \varphi(x)$	Assumption
2.	$\mathbb{T} \exists x \varphi(x)$	$\rightarrow \mathbb{F}$ 1
3.	$\mathbb{F} \forall x \varphi(x)$	$\rightarrow \mathbb{F}$ 1
4.	$\mathbb{T} \varphi(a)$	$\exists \mathbb{T}$ 2
5.	$\mathbb{F} \varphi(a)$	$\forall \mathbb{F}$ 3
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However, $\exists x \varphi(x) \rightarrow \forall x \varphi(x)$ is not valid.

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Bibliography