

## tab.1 Derivability and the Quantifiers

fol:tab:qpr:sec The completeness theorem also requires that the tableaux rules yield the facts explanation about  $\vdash$  established in this section.

fol:tab:qpr:thm:strong-generalization **Theorem tab.1.** *If  $c$  is a constant not occurring in  $\Gamma$  or  $\varphi(x)$  and  $\Gamma \vdash \varphi(c)$ , then  $\Gamma \vdash \forall x \varphi(x)$ .*

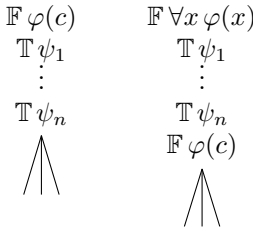
*Proof.* Suppose  $\Gamma \vdash \varphi(c)$ , i.e., there are  $\psi_1, \dots, \psi_n \in \Gamma$  and a closed **tableau** for

$$\{\mathbb{F} \varphi(c), \mathbb{T} \psi_1, \dots, \mathbb{T} \psi_n\}.$$

We have to show that there is also a closed **tableau** for

$$\{\mathbb{F} \forall x \varphi(x), \mathbb{T} \psi_1, \dots, \mathbb{T} \psi_n\}.$$

Take the closed **tableau** and replace the first assumption with  $\mathbb{F} \forall x \varphi(x)$ , and insert  $\mathbb{F} \varphi(c)$  after the assumptions.



The tableau is still closed, since all **sentences** available as assumptions before are still available at the top of the **tableau**. The inserted line is the result of a correct application of  $\forall\mathbb{F}$ , since the **constant symbol**  $c$  does not occur in  $\psi_1, \dots, \psi_n$  or  $\forall x \varphi(x)$ , i.e., it does not occur above the inserted line in the new **tableau**.  $\square$

fol:tab:qpr:prop:provability-quantifiers

**Proposition tab.2.**

1.  $\varphi(t) \vdash \exists x \varphi(x)$ .
2.  $\forall x \varphi(x) \vdash \varphi(t)$ .

*Proof.* 1. A closed **tableau** for  $\mathbb{F} \exists x \varphi(x), \mathbb{T} \varphi(t)$  is:

- |    |                                   |                       |
|----|-----------------------------------|-----------------------|
| 1. | $\mathbb{F} \exists x \varphi(x)$ | Assumption            |
| 2. | $\mathbb{T} \varphi(t)$           | Assumption            |
| 3. | $\mathbb{F} \varphi(t)$           | $\exists\mathbb{F} 1$ |
|    | $\otimes$                         |                       |

2. A closed **tableau** for  $\mathbb{F} \varphi(t), \mathbb{T} \forall x \varphi(x)$ , is:

1.  $\mathbb{F} \varphi(t)$  Assumption
2.  $\mathbb{T} \forall x \varphi(x)$  Assumption
3.  $\mathbb{T} \varphi(t)$   $\forall \mathbb{T} 2$   
 $\otimes$

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## Bibliography