

tab.1 Derivability and the Quantifiers

fol:tab:qpr:
 fol:tab:qpr:^{sec}
 thm:strong-generalization

Theorem tab.1. *If c is a constant not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x \varphi(x)$.*

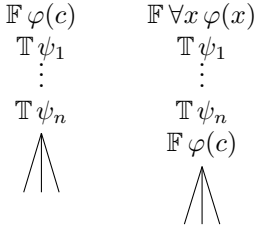
Proof. Suppose $\Gamma \vdash \varphi(c)$, i.e., there are $\psi_1, \dots, \psi_n \in \Gamma$ and a closed **tableau** for

$$\{\mathbb{F} \varphi(c), \mathbb{T} \psi_1, \dots, \mathbb{T} \psi_n\}.$$

We have to show that there is also a closed **tableau** for

$$\{\mathbb{F} \forall x \varphi(x), \mathbb{T} \psi_1, \dots, \mathbb{T} \psi_n\}.$$

Take the closed **tableau** and replace the first assumption with $\mathbb{F} \forall x \varphi(x)$, and insert $\mathbb{F} \varphi(c)$ after the assumptions.



The tableau is still closed, since all **sentences** available as assumptions before are still available at the top of the **tableau**. The inserted line is the result of a correct application of $\forall\mathbb{F}$, since the **constant symbol** c does not occur in ψ_1, \dots, ψ_n of $\forall x \varphi(x)$, i.e., it does not occur above the inserted line in the new **tableau**. \square

fol:tab:qpr:
 prop:provability-quantifiers

Proposition tab.2.

1. $\varphi(t) \vdash \exists x \varphi(x)$.

2. $\forall x \varphi(x) \vdash \varphi(t)$.

Proof. 1. A closed **tableau** for $\mathbb{F} \exists x \varphi(x), \mathbb{T} \varphi(t)$ is:

- | | | |
|----|-----------------------------------|-----------------------|
| 1. | $\mathbb{F} \exists x \varphi(x)$ | Assumption |
| 2. | $\mathbb{T} \varphi(t)$ | Assumption |
| 3. | $\mathbb{F} \varphi(t)$ | $\exists\mathbb{F} 1$ |
| | \otimes | |

2. A closed **tableau** for $\mathbb{F} \varphi(t), \mathbb{T} \forall x \varphi(x)$, is:

1. $\mathbb{F} \varphi(t)$ Assumption
2. $\mathbb{T} \forall x \varphi(x)$ Assumption
3. $\mathbb{T} \varphi(t)$ $\forall \mathbb{T} 2$
 \otimes

Photo Credits

Bibliography