

syn.1 Main operator of a Formula

fol:syn:mai: It is often useful to talk about the last operator used in constructing a formula φ . This operator is called the *main operator* of φ . Intuitively, it is the “outermost” operator of φ . For example, the main operator of $\neg\varphi$ is \neg , the main operator of $(\varphi \vee \psi)$ is \vee , etc. explanation sec

fol:syn:mai: **Definition syn.1 (Main operator).** The *main operator* of a formula φ is defined as follows: def:main-op

1. φ is atomic: φ has no main operator.
2. $\varphi \equiv \neg\psi$: the main operator of φ is \neg .
3. $\varphi \equiv (\psi \wedge \chi)$: the main operator of φ is \wedge .
4. $\varphi \equiv (\psi \vee \chi)$: the main operator of φ is \vee .
5. $\varphi \equiv (\psi \rightarrow \chi)$: the main operator of φ is \rightarrow .
6. $\varphi \equiv (\psi \leftrightarrow \chi)$: the main operator of φ is \leftrightarrow .
7. $\varphi \equiv \forall x \psi$: the main operator of φ is \forall .
8. $\varphi \equiv \exists x \psi$: the main operator of φ is \exists .

In each case, we intend the specific indicated *occurrence* of the main operator in the formula. For instance, since the formula $((\theta \rightarrow \alpha) \rightarrow (\alpha \rightarrow \theta))$ is of the form $(\psi \rightarrow \chi)$ where ψ is $(\theta \rightarrow \alpha)$ and χ is $(\alpha \rightarrow \theta)$, the second occurrence of \rightarrow is the main operator.

This is a *recursive* definition of a function which maps all non-atomic formulas to their main operator occurrence. Because of the way formulas are defined inductively, every formula φ satisfies one of the cases in Definition syn.1. This guarantees that for each non-atomic formula φ a main operator exists. Because each formula satisfies only one of these conditions, and because the smaller formulas from which φ is constructed are uniquely determined in each case, the main operator occurrence of φ is unique, and so we have defined a function. explanation

We call formulas by the following names depending on which symbol their main operator is:

| Main operator | Type of formula | Example |
|---------------|-----------------------|--|
| none | atomic (formula) | $\perp, \top, R(t_1, \dots, t_n), t_1 = t_2$ |
| \neg | negation | $\neg\varphi$ |
| \wedge | conjunction | $(\varphi \wedge \psi)$ |
| \vee | disjunction | $(\varphi \vee \psi)$ |
| \rightarrow | conditional | $(\varphi \rightarrow \psi)$ |
| \forall | universal (formula) | $\forall x \varphi$ |
| \exists | existential (formula) | $\exists x \varphi$ |

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Bibliography