

syn.1 Introduction

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Giving the meaning of expressions is the domain of semantics. The central concept in semantics is that of satisfaction in a **structure**. A **structure** gives meaning to the building blocks of the language: a **domain** is a non-empty set of objects. The quantifiers are interpreted as ranging over this domain, **constant symbols** are assigned elements in the domain, **function symbols** are assigned functions from the **domain** to itself, and **predicate symbols** are assigned relations on the **domain**. The **domain** together with assignments to the basic vocabulary constitutes a **structure**. **Variables** may appear in **formulas**, and in order to give a semantics, we also have to assign **elements** of the **domain** to them—this is a variable assignment. The satisfaction relation, finally, brings these together. A **formula** may be satisfied in a **structure** \mathfrak{M} relative to a **variable** assignment s , written as $\mathfrak{M}, s \models \varphi$. This relation is also defined by induction on the structure of φ , using the truth tables for the logical connectives to define, say, satisfaction of $(\varphi \wedge \psi)$ in terms of satisfaction (or not) of φ and ψ . It then turns out that the **variable** assignment is irrelevant if the **formula** φ is a **sentence**, i.e., has no free variables, and so we can talk of **sentences** being simply satisfied (or not) in **structures**.

On the basis of the satisfaction relation $\mathfrak{M} \models \varphi$ for **sentences** we can then define the basic semantic notions of validity, entailment, and satisfiability. A **sentence** is valid, $\models \varphi$, if every **structure** satisfies it. It is entailed by a set of **sentences**, $\Gamma \models \varphi$, if every **structure** that satisfies all the **sentences** in Γ also satisfies φ . And a set of **sentences** is satisfiable if some **structure** satisfies all **sentences** in it at the same time. Because **formulas** are inductively defined, and satisfaction is in turn defined by induction on the structure of **formulas**, we can use induction to prove properties of our semantics and to relate the semantic notions defined.

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Bibliography