

syn.1 Free Variables and Sentences

fol:syn:fvs:
 fol:syn:fvs:^{sec}
 defn:free-occ

Definition syn.1 (Free occurrences of a variable). The *free* occurrences of a **variable** in a **formula** are defined inductively as follows:

1. φ is atomic: all **variable** occurrences in φ are free.
2. $\varphi \equiv \neg\psi$: the free **variable** occurrences of φ are exactly those of ψ .
3. $\varphi \equiv (\psi * \chi)$: the free **variable** occurrences of φ are those in ψ together with those in χ .
4. $\varphi \equiv \forall x \psi$: the free **variable** occurrences in φ are all of those in ψ except for occurrences of x .
5. $\varphi \equiv \exists x \psi$: the free **variable** occurrences in φ are all of those in ψ except for occurrences of x .

Definition syn.2 (Bound Variables). An occurrence of a **variable** in a formula φ is *bound* if it is not free.

Problem syn.1. Give an inductive definition of the bound variable occurrences along the lines of **Definition syn.1**.

Definition syn.3 (Scope). If $\forall x \psi$ is an occurrence of a subformula in a formula φ , then the corresponding occurrence of ψ in φ is called the *scope* of the corresponding occurrence of $\forall x$. Similarly for $\exists x$.

If ψ is the scope of a quantifier occurrence $\forall x$ or $\exists x$ in φ , then the free occurrences of x in ψ are bound in $\forall x \psi$ and $\exists x \psi$. We say that these occurrences are *bound by* the mentioned quantifier occurrence.

Example syn.4. Consider the following formula:

$$\exists v_0 \underbrace{A_0^2(v_0, v_1)}_{\psi}$$

ψ represents the scope of $\exists v_0$. The quantifier binds the occurrence of v_0 in ψ , but does not bind the occurrence of v_1 . So v_1 is a free variable in this case.

We can now see how this might work in a more complicated **formula** φ :

$$\forall v_0 \underbrace{(A_0^1(v_0) \rightarrow A_0^2(v_0, v_1))}_{\psi} \rightarrow \exists v_1 \underbrace{(A_1^2(v_0, v_1) \vee \forall v_0 \underbrace{\neg A_1^1(v_0)}_{\theta})}_{\chi}$$

ψ is the scope of the first $\forall v_0$, χ is the scope of $\exists v_1$, and θ is the scope of the second $\forall v_0$. The first $\forall v_0$ binds the occurrences of v_0 in ψ , $\exists v_1$ the occurrence of v_1 in χ , and the second $\forall v_0$ binds the occurrence of v_0 in θ . The first occurrence of v_1 and the fourth occurrence of v_0 are free in φ . The last occurrence of v_0 is free in θ , but bound in χ and φ .

Definition syn.5 (Sentence). A **formula** φ is a *sentence* iff it contains no free occurrences of **variables**.

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Bibliography