

## syn.1 Covered Structures for First-order Languages

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sec

Recall that a term is *closed* if it contains no **variables**.

explanation

**Definition syn.1 (Value of closed terms).** If  $t$  is a closed term of the language  $\mathcal{L}$  and  $\mathfrak{M}$  is a **structure** for  $\mathcal{L}$ , the **value**  $\text{Val}^{\mathfrak{M}}(t)$  is defined as follows:

1. If  $t$  is just the **constant symbol**  $c$ , then  $\text{Val}^{\mathfrak{M}}(c) = c^{\mathfrak{M}}$ .
2. If  $t$  is of the form  $f(t_1, \dots, t_n)$ , then

$$\text{Val}^{\mathfrak{M}}(t) = f^{\mathfrak{M}}(\text{Val}^{\mathfrak{M}}(t_1), \dots, \text{Val}^{\mathfrak{M}}(t_n)).$$

**Definition syn.2 (Covered structure).** A **structure** is *covered* if every element of the domain is the **value** of some closed term.

**Example syn.3.** Let  $\mathcal{L}$  be the language with **constant symbols** *zero*, *one*, *two*,  $\dots$ , the binary **predicate symbol**  $<$ , and the binary **function symbols**  $+$  and  $\times$ . Then a **structure**  $\mathfrak{M}$  for  $\mathcal{L}$  is the one with domain  $|\mathfrak{M}| = \{0, 1, 2, \dots\}$  and assignments  $zero^{\mathfrak{M}} = 0$ ,  $one^{\mathfrak{M}} = 1$ ,  $two^{\mathfrak{M}} = 2$ , and so forth. For the binary relation symbol  $<$ , the set  $<^{\mathfrak{M}}$  is the set of all pairs  $\langle c_1, c_2 \rangle \in |\mathfrak{M}|^2$  such that  $c_1$  is less than  $c_2$ : for example,  $\langle 1, 3 \rangle \in <^{\mathfrak{M}}$  but  $\langle 2, 2 \rangle \notin <^{\mathfrak{M}}$ . For the binary **function symbol**  $+$ , define  $+^{\mathfrak{M}}$  in the usual way—for example,  $+^{\mathfrak{M}}(2, 3)$  maps to 5, and similarly for the binary **function symbol**  $\times$ . Hence, the **value** of *four* is just 4, and the **value** of  $\times(\textit{two}, +(\textit{three}, \textit{zero}))$  (or in infix notation,  $\textit{two} \times (\textit{three} + \textit{zero})$ ) is

$$\begin{aligned} \text{Val}^{\mathfrak{M}}(\times(\textit{two}, +(\textit{three}, \textit{zero}))) &= \\ &= \times^{\mathfrak{M}}(\text{Val}^{\mathfrak{M}}(\textit{two}), \text{Val}^{\mathfrak{M}}(+(\textit{three}, \textit{zero}))) \\ &= \times^{\mathfrak{M}}(\text{Val}^{\mathfrak{M}}(\textit{two}), +^{\mathfrak{M}}(\text{Val}^{\mathfrak{M}}(\textit{three}), \text{Val}^{\mathfrak{M}}(\textit{zero}))) \\ &= \times^{\mathfrak{M}}(\textit{two}^{\mathfrak{M}}, +^{\mathfrak{M}}(\textit{three}^{\mathfrak{M}}, \textit{zero}^{\mathfrak{M}})) \\ &= \times^{\mathfrak{M}}(2, +^{\mathfrak{M}}(3, 0)) \\ &= \times^{\mathfrak{M}}(2, 3) \\ &= 6 \end{aligned}$$

**Problem syn.1.** Is  $\mathfrak{N}$ , the standard model of arithmetic, covered? Explain.

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Bibliography