We also need a few rules that allow us to rearrange sentences in the left and right side of a sequent. Since the logical rules require that the sentences in the premise which the rule acts upon stand either to the far left or to the far right, we need an “exchange” rule that allows us to move sentences to the right position. It’s also important sometimes to be able to combine two identical sentences into one, and to add a sentence on either side.

Weakening

\[
\frac{\Gamma \Rightarrow \Delta}{\varphi, \Gamma \Rightarrow \Delta} \quad \text{WL} \\
\frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, \varphi} \quad \text{WR}
\]

Contraction

\[
\frac{\varphi, \varphi, \Gamma \Rightarrow \Delta}{\varphi, \Gamma \Rightarrow \Delta} \quad \text{CL} \\
\frac{\Gamma \Rightarrow \Delta, \varphi, \varphi}{\Gamma \Rightarrow \Delta, \varphi} \quad \text{CR}
\]

Exchange

\[
\frac{\Gamma, \varphi, \psi, \Pi \Rightarrow \Delta}{\Gamma, \psi, \varphi, \Pi \Rightarrow \Delta} \quad \text{XL} \\
\frac{\Gamma \Rightarrow \Delta, \varphi, \psi, A}{\Gamma \Rightarrow \Delta, \psi, \varphi, A} \quad \text{XR}
\]

A series of weakening, contraction, and exchange inferences will often be indicated by double inference lines.

The following rule, called “cut,” is not strictly speaking necessary, but makes it a lot easier to reuse and combine derivations.

\[
\frac{\varphi, \Pi \Rightarrow \Delta, \varphi, \Pi \Rightarrow A}{\Gamma, \Pi \Rightarrow \Delta, A} \quad \text{Cut}
\]