

seq.1 Rules and Derivations

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sec For the following, let $\Gamma, \Delta, \Pi, \Lambda$ represent finite sequences of **sentences**.

Definition seq.1 (Sequent). A *sequent* is an expression of the form

$$\Gamma \Rightarrow \Delta$$

where Γ and Δ are finite (possibly empty) sequences of **sentences** of the language \mathcal{L} . Γ is called the *antecedent*, while Δ is the *succedent*.

The intuitive idea behind a sequent is: if all of the **sentences** in the antecedent hold, then at least one of the **sentences** in the succedent holds. That is, if $\Gamma = \langle \varphi_1, \dots, \varphi_m \rangle$ and $\Delta = \langle \psi_1, \dots, \psi_n \rangle$, then $\Gamma \Rightarrow \Delta$ holds iff explanation

$$(\varphi_1 \wedge \dots \wedge \varphi_m) \rightarrow (\psi_1 \vee \dots \vee \psi_n)$$

holds. There are two special cases: where Γ is empty and when Δ is empty. When Γ is empty, i.e., $m = 0$, $\Rightarrow \Delta$ holds iff $\psi_1 \vee \dots \vee \psi_n$ holds. When Δ is empty, i.e., $n = 0$, $\Gamma \Rightarrow$ holds iff $\neg(\varphi_1 \wedge \dots \wedge \varphi_m)$ does. We say a sequent is valid iff the corresponding **sentence** is valid.

If Γ is a sequence of **sentences**, we write Γ, φ for the result of appending φ to the right end of Γ (and φ, Γ for the result of appending φ to the left end of Γ). If Δ is a sequence of **sentences** also, then Γ, Δ is the concatenation of the two sequences.

Definition seq.2 (Initial Sequent). An *initial sequent* is a sequent of one of the following forms:

1. $\varphi \Rightarrow \varphi$
2. $\Rightarrow \top$
3. $\perp \Rightarrow$

for any **sentence** φ in the language.

Derivations in the sequent calculus are certain trees of sequents, where the topmost sequents are initial sequents, and if a sequent stands below one or two other sequents, it must follow correctly by a rule of inference. The rules for **LK** are divided into two main types: *logical* rules and *structural* rules. The logical rules are named for the **main operator** of the **sentence** containing φ and/or ψ in the lower sequent. Each one comes in two versions, one for inferring a sequent with the **sentence** containing the **logical operator** on the left, and one with the **sentence** on the right.

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Bibliography