

## seq.1 Quantifier Rules

fol:seq:qrl:  
sec **Rules for  $\forall$**

$$\frac{\varphi(t), \Gamma \Rightarrow \Delta}{\forall x \varphi(x), \Gamma \Rightarrow \Delta} \forall L \qquad \frac{\Gamma \Rightarrow \Delta, \varphi(a)}{\Gamma \Rightarrow \Delta, \forall x \varphi(x)} \forall R$$

In  $\forall L$ ,  $t$  is a closed term (i.e., one without variables). In  $\forall R$ ,  $a$  is a **constant symbol** which must not occur anywhere in the lower sequent of the  $\forall R$  rule. We call  $a$  the *eigenvariable* of the  $\forall R$  inference.

**Rules for  $\exists$**

$$\frac{\varphi(a), \Gamma \Rightarrow \Delta}{\exists x \varphi(x), \Gamma \Rightarrow \Delta} \exists L \qquad \frac{\Gamma \Rightarrow \Delta, \varphi(t)}{\Gamma \Rightarrow \Delta, \exists x \varphi(x)} \exists R$$

Again,  $t$  is a closed term, and  $a$  is a **constant symbol** which does not occur in the lower sequent of the  $\exists L$  rule. We call  $a$  the *eigenvariable* of the  $\exists L$  inference.

The condition that an eigenvariable not occur in the lower sequent of the  $\forall R$  or  $\exists L$  inference is called the *eigenvariable condition*.

We use the term “eigenvariable” even though  $a$  in the above rules is a **constant symbol**. This has historical reasons. explanation

In  $\exists R$  and  $\forall L$  there are no restrictions on the term  $t$ . On the other hand, in the  $\exists L$  and  $\forall R$  rules, the eigenvariable condition requires that the **constant symbol**  $a$  does not occur anywhere outside of  $\varphi(a)$  in the upper sequent. It is necessary to ensure that the system is sound, i.e., only **derives** sequents that are valid. Without this condition, the following would be allowed:

$$\frac{\varphi(a) \Rightarrow \varphi(a)}{\exists x \varphi(x) \Rightarrow \varphi(a)} * \exists L \qquad \frac{\varphi(a) \Rightarrow \varphi(a)}{\varphi(a) \Rightarrow \forall x \varphi(x)} * \forall R$$
$$\frac{\varphi(a) \Rightarrow \varphi(a)}{\exists x \varphi(x) \Rightarrow \forall x \varphi(x)} \forall R \qquad \frac{\varphi(a) \Rightarrow \forall x \varphi(x)}{\exists x \varphi(x) \Rightarrow \forall x \varphi(x)} \exists L$$

However,  $\exists x \varphi(x) \Rightarrow \forall x \varphi(x)$  is not valid.

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**Bibliography**