

ntd.1 Propositional Rules

fol:ntd:prl:
sec **Rules for \wedge**

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \wedge\text{Intro} \qquad \frac{\varphi \wedge \psi}{\varphi} \wedge\text{Elim}$$

$$\frac{\varphi \wedge \psi}{\psi} \wedge\text{Elim}$$

Rules for \vee

$$\frac{\varphi}{\varphi \vee \psi} \vee\text{Intro}$$

$$\frac{\psi}{\varphi \vee \psi} \vee\text{Intro}$$

$$n \frac{\varphi \vee \psi \quad \begin{array}{c} [\varphi]^n \\ \vdots \\ \chi \end{array} \quad \begin{array}{c} [\psi]^n \\ \vdots \\ \chi \end{array}}{\chi} \vee\text{Elim}$$

Rules for \rightarrow

$$n \frac{\begin{array}{c} [\varphi]^n \\ \vdots \\ \psi \end{array}}{\varphi \rightarrow \psi} \rightarrow\text{Intro}$$

$$\frac{\varphi \rightarrow \psi \quad \varphi}{\psi} \rightarrow\text{Elim}$$

Rules for \neg

$$n \frac{\begin{array}{c} [\varphi]^n \\ \vdots \\ \perp \end{array}}{\neg\varphi} \neg\text{Intro}$$

$$\frac{\neg\varphi \quad \varphi}{\perp} \neg\text{Elim}$$

Rules for \perp

$$\frac{\perp}{\varphi} \perp_I \qquad \begin{array}{c} [\neg\varphi]^n \\ \vdots \\ n \frac{\perp}{\varphi} \perp_C \end{array}$$

Note that \neg -Intro and \perp_C are very similar: The difference is that \neg -Intro derives a negated **sentence** $\neg\varphi$ but \perp_C a positive **sentence** φ .

Whenever a rule indicates that some assumption may be discharged, we take this to be a permission, but not a requirement. E.g., in the \rightarrow -Intro rule, we may discharge any number of assumptions of the form φ in the **derivation** of the premise ψ , including zero.

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Bibliography