### Propositional Rules

#### Rules for $\land$

- **$\land$Intro**
  \[
  \varphi, \psi \vdash \varphi \land \psi
  \]

- **$\land$Elim**
  \[
  \frac{\varphi \land \psi}{\varphi} \\
  \frac{\varphi \land \psi}{\psi}
  \]

#### Rules for $\lor$

- **$\lor$Intro**
  \[
  \frac{\varphi}{\varphi \lor \psi} \\
  \frac{\psi}{\varphi \lor \psi}
  \]

- **$\lor$Elim**
  \[
  \frac{\varphi \lor \psi}{\chi} \quad \frac{\varphi \lor \psi}{\chi} \\
  \frac{\varphi \lor \psi}{\chi}
  \]

#### Rules for $\rightarrow$

- **$\rightarrow$Intro**
  \[
  \frac{\varphi \rightarrow \psi}{\psi} \\
  \frac{\varphi \rightarrow \psi}{\psi}
  \]

- **$\rightarrow$Elim**
  \[
  \frac{\varphi \land \psi}{\varphi} \\
  \frac{\varphi \land \psi}{\psi}
  \]

#### Rules for $\neg$

- **$\neg$Intro**
  \[
  \frac{\varphi}{\neg \varphi}
  \]

- **$\neg$Elim**
  \[
  \frac{\neg \varphi}{\bot}
  \]

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Rules for $\bot$

\[
\frac{\bot}{\phi} \quad \bot_I
\]

Note that $\neg$Intro and $\bot_C$ are very similar: The difference is that $\neg$Intro derives a negated sentence $\neg \phi$ but $\bot_C$ a positive sentence $\phi$.

Whenever a rule indicates that some assumption may be discharged, we take this to be a permission, but not a requirement. E.g., in the $\rightarrow$Intro rule, we may discharge any number of assumptions of the form $\phi$ in the derivation of the premise $\psi$, including zero.

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Bibliography