axd.1 Derivations with Quantifiers

fol:axd:prq:sec

Example axd.1. Let us give a derivation of $(\forall x\, \varphi(x) \land \forall y\, \psi(y)) \rightarrow \forall x\, (\varphi(x) \land \psi(x))$.

First, note that

$$(\forall x\, \varphi(x) \land \forall y\, \psi(y)) \rightarrow \forall x\, \varphi(x)$$

is an instance of **??**, and

$$\forall x\, \varphi(x) \rightarrow \varphi(a)$$

of **??**. So, by **??**, we know that

$$(\forall x\, \varphi(x) \land \forall y\, \psi(y)) \rightarrow \varphi(a)$$

is derivable. Likewise, since

$$\begin{aligned} (\forall x\, \varphi(x) \land \forall y\, \psi(y)) \rightarrow \forall y\, \psi(y) \qquad &\text{and} \\ \forall y\, \psi(y) \rightarrow \psi(a) \end{aligned}$$

are instances of **??** and **??**, respectively,

$$(\forall x\, \varphi(x) \land \forall y\, \psi(y)) \rightarrow \psi(a)$$

is derivable by **??**. Using an appropriate instance of **??** and two applications of MP, we see that

$$(\forall x\, \varphi(x) \land \forall y\, \psi(y)) \rightarrow (\varphi(a) \land \psi(a))$$

is derivable. We can now apply QR to obtain

$$(\forall x\, \varphi(x) \land \forall y\, \psi(y)) \rightarrow \forall x\, (\varphi(x) \land \psi(x)).$$

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Bibliography