axd.1 Derivability and the Quantifiers

fol:axd:qpr:sec

The completeness theorem also requires that axiomatic deductions yield the facts about $\vdash$ established in this section. explanation

fol:axd:qpr:thm:strong-generalization

Theorem axd.1. *If c is a constant symbol not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x\, \varphi(x)$.*

Proof. By the deduction theorem, $\Gamma \vdash \top \to \varphi(c)$. Since c does not occur in Γ or $\top$, we get $\Gamma \vdash \top \to \varphi(c)$. By the deduction theorem again, $\Gamma \vdash \forall x\, \varphi(x)$. $\square$

fol:axd:qpr:prop:provability-quantifiers

Proposition axd.2.

1. $\varphi(t) \vdash \exists x\, \varphi(x)$.
2. $\forall x\, \varphi(x) \vdash \varphi(t)$.

Proof.
1. By **??** and the deduction theorem.
2. By **??** and the deduction theorem. $\square$

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Bibliography