

axd.1 Derivability and the Quantifiers

fol:axd:qpr:

fol:axd:qpr:^{sec}

thm:strong-generalization

Theorem axd.1. *If c is a constant symbol not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x \varphi(x)$.*

Proof. By the deduction theorem, $\Gamma \vdash \top \rightarrow \varphi(c)$. Since c does not occur in Γ or \top , we get $\Gamma \vdash \top \rightarrow \varphi(c)$. By the deduction theorem again, $\Gamma \vdash \forall x \varphi(x)$. \square

fol:axd:qpr:

prop:provability-quantifiers

Proposition axd.2.

1. $\varphi(t) \vdash \exists x \varphi(x)$.
2. $\forall x \varphi(x) \vdash \varphi(t)$.

Proof. 1. By ?? and the deduction theorem.

2. By ?? and the deduction theorem. \square

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Bibliography