

## axd.1 Derivability and the Propositional Connectives

fol:axd:ppr:sec We establish that the **derivability** relation  $\vdash$  of axiomatic deduction is strong explanation enough to establish some basic facts involving the propositional connectives, such as that  $\varphi \wedge \psi \vdash \varphi$  and  $\varphi, \varphi \rightarrow \psi \vdash \psi$  (modus ponens). These facts are needed for the proof of the completeness theorem.

### Proposition axd.1.

fol:axd:ppr:prop:provability-land  
fol:axd:ppr:prop:provability-land-left  
fol:axd:ppr:prop:provability-land-right

1. Both  $\varphi \wedge \psi \vdash \varphi$  and  $\varphi \wedge \psi \vdash \psi$
2.  $\varphi, \psi \vdash \varphi \wedge \psi$ .

*Proof.* 1. From ?? and ?? by modus ponens.

2. From ?? by two applications of modus ponens. □

### Proposition axd.2.

fol:axd:ppr:prop:provability-lor

1.  $\varphi \vee \psi, \neg\varphi, \neg\psi$  is inconsistent.
2. Both  $\varphi \vdash \varphi \vee \psi$  and  $\psi \vdash \varphi \vee \psi$ .

*Proof.* 1. From ?? we get  $\vdash \neg\varphi \rightarrow (\varphi \rightarrow \perp)$  and  $\vdash \neg\psi \rightarrow (\psi \rightarrow \perp)$ . So by the deduction theorem, we have  $\{\neg\varphi\} \vdash \varphi \rightarrow \perp$  and  $\{\neg\psi\} \vdash \psi \rightarrow \perp$ . From ?? we get  $\{\neg\varphi, \neg\psi\} \vdash (\varphi \vee \psi) \rightarrow \perp$ . By the deduction theorem,  $\{\varphi \vee \psi, \neg\varphi, \neg\psi\} \vdash \perp$ .

2. From ?? and ?? by modus ponens. □

### Proposition axd.3.

fol:axd:ppr:prop:provability-lif  
fol:axd:ppr:prop:provability-lif-left  
fol:axd:ppr:prop:provability-lif-right

1.  $\varphi, \varphi \rightarrow \psi \vdash \psi$ .
2. Both  $\neg\varphi \vdash \varphi \rightarrow \psi$  and  $\psi \vdash \varphi \rightarrow \psi$ .

*Proof.* 1. We can **derive**:

1.  $\varphi$            HYP
2.  $\varphi \rightarrow \psi$    HYP
3.  $\psi$             1, 2, MP

2. By ?? and ?? and the deduction theorem, respectively. □

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## Bibliography