We establish that the derivability relation $\vdash$ of axiomatic deduction is strong enough to establish some basic facts involving the propositional connectives, such as that $\varphi \land \psi \vdash \varphi$ and $\varphi, \varphi \rightarrow \psi \vdash \psi$ (modus ponens). These facts are needed for the proof of the completeness theorem.

**Proposition axd.1.**

1. Both $\varphi \land \psi \vdash \varphi$ and $\varphi \land \psi \vdash \psi$
2. $\varphi, \psi \vdash \varphi \land \psi$.

*Proof.*** 1. From ?? and ?? by modus ponens.
2. From ?? by two applications of modus ponens.

**Proposition axd.2.**

1. $\varphi \lor \psi, \neg \varphi, \neg \psi$ is inconsistent.
2. Both $\varphi \vdash \varphi \lor \psi$ and $\psi \vdash \varphi \lor \psi$.

*Proof.*** 1. From ?? we get $\vdash \neg \varphi \rightarrow (\varphi \rightarrow \bot)$ and $\vdash \neg \psi \rightarrow (\psi \rightarrow \bot)$. So by the deduction theorem, we have $\{\neg \varphi\} \vdash \varphi \rightarrow \bot$ and $\{\neg \psi\} \vdash \psi \rightarrow \bot$. From ?? we get $\{\neg \varphi, \neg \psi\} \vdash (\varphi \lor \psi) \rightarrow \bot$. By the deduction theorem, $\{\varphi \lor \psi, \neg \varphi, \neg \psi\} \vdash \bot$.
2. From ?? and ?? by modus ponens.

**Proposition axd.3.**

1. $\varphi, \varphi \rightarrow \psi \vdash \psi$.
2. Both $\neg \varphi \vdash \varphi \rightarrow \psi$ and $\psi \vdash \varphi \rightarrow \psi$.

*Proof.*** 1. We can derive:

1. $\varphi$ HYP
2. $\varphi \rightarrow \psi$ HYP
3. $\psi$ 1, 2, MP

2. By ?? and ?? and the deduction theorem, respectively.

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**Bibliography**