

axd.1 Derivations with Identity predicate

fol:axd:ide:sec In order to accommodate = in **derivations**, we simply add new axiom schemas. The definition of **derivation** and \vdash remains the same, we just also allow the new axioms.

Definition axd.1 (Axioms for identity predicate).

fol:axd:ide:ax:id1 $t = t,$ (1)

fol:axd:ide:ax:id2 $t_1 = t_2 \rightarrow (\psi(t_1) \rightarrow \psi(t_2)),$ (2)

for any closed terms t, t_1, t_2 .

fol:axd:ide:prop:sound **Proposition axd.2.** *The axioms eq. (1) and eq. (2) are valid.*

Proof. Exercise. □

Problem axd.1. Prove **Proposition axd.2**.

fol:axd:ide:prop:iden1 **Proposition axd.3.** $\Gamma \vdash t = t,$ for any term t and set Γ .

fol:axd:ide:prop:iden2 **Proposition axd.4.** If $\Gamma \vdash \varphi(t_1)$ and $\Gamma \vdash t_1 = t_2,$ then $\Gamma \vdash \varphi(t_2).$

Proof. The **formula** $(t_1 = t_2 \rightarrow (\varphi(t_1) \rightarrow \varphi(t_2)))$

is an instance of **eq. (2)**. The conclusion follows by two applications of MP. □

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Bibliography