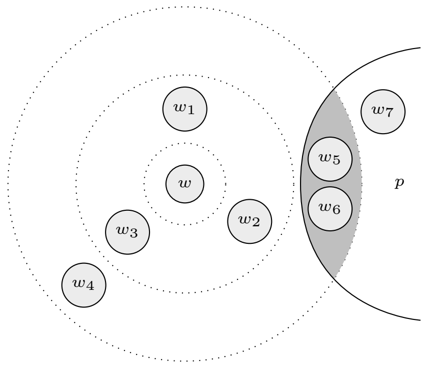

Figure 1: Diagram of a sphere model

con:min:sph:
fig:sphere-model

min.1 Sphere Models

con:min:sph:
sec

One way of providing a formal semantics for counterfactuals is to turn Lewis's informal account into a mathematical structure. The spheres around a world w then are sets of worlds. Since the spheres are nested, the sets of worlds around w have to be linearly ordered by the subset relation.

Definition min.1. A *sphere model* is a triple $\mathfrak{M} = \langle W, O, V \rangle$ where W is a non-empty set of worlds, $V \colon \mathrm{At}_0 \to \wp(W)$ is a valuation, and $O \colon W \to \wp(\wp(W))$ assigns to each world w a *system of spheres* O_w. For each w, O_w is a set of sets of worlds, and must satisfy:

1. O_w is *centered* on w: $\{w\} \in O_w$.
2. O_w is *nested*: whenever S_1, $S_2 \in O_w$, $S_1 \subseteq S_2$ or $S_2 \subseteq S_1$, i.e., O_w is linearly ordered by $\subseteq$.
3. O_w is closed under non-empty unions.
4. O_w is closed under non-empty intersections.

The intuition behind O_w is that the worlds "around" w are stratified according to how far away they are from w. The innermost sphere is just w by itself, i.e., the set $\{w\}$: w is closer to w than the worlds in any other sphere. If $S \subsetneq S'$, then the worlds in $S' \setminus S$ are further way from w than the worlds in S: $S' \setminus S$ is the "layer" between the S and the worlds outside of S'. In particular, we have to think of the spheres as containing all the worlds within their outer surface; they are not just the individual layers.

The diagram in Figure 1 corresponds to the sphere model with $W = \{w, w_1, \dots, w_7\}$, $V(p) = \{w_5, w_6, w_7\}$. The innermost sphere $S_1 = \{w\}$. The closest worlds to w are w_1, w_2, w_3, so the next larger sphere is $S_2 = \{w, w_1, w_2, w_3\}$.

The worlds further out are w_4, w_5, w_6, so the outermost sphere is $S_3 = \{w, w_1, \dots, w_6\}$. The system of spheres around w is $O_w = \{S_1, S_2, S_3\}$. The world w_7 is not in any sphere around w. The closest worlds in which p is true are w_5 and w_6, and so the smallest p-admitting sphere is S_3.

To define satisfaction of a formula φ at world w in a sphere model $\mathfrak{M}$, $\mathfrak{M}, w \Vdash \varphi$, we expand the definition for modal formulas to include a clause for $\psi \mathrel{\Box\!\!\rightarrow} \chi$:

Definition min.2. $\mathfrak{M}, w \Vdash \psi \mathrel{\Box\!\!\rightarrow} \chi$ iff either

1. For all $u \in \bigcup O_w$, $\mathfrak{M}, u \nVdash \psi$, or con:min:sph: sphere-vac
2. For some $S \in O_w$, con:min:sph: sphere-nonvac
   a) $\mathfrak{M}, u \Vdash \psi$ for some $u \in S$, and
   b) for all $v \in S$, either $\mathfrak{M}, v \nVdash \psi$ or $\mathfrak{M}, v \Vdash \chi$.

According to this definition, $\mathfrak{M}, w \Vdash \psi \mathrel{\Box\!\!\rightarrow} \chi$ iff either the antecedent ψ is false everywhere in the spheres around w, or there is a sphere S where ψ is true, and the material conditional $\psi \rightarrow \chi$ is true at all worlds in that "ψ-admitting" sphere. Note that we didn't require in the definition that S is the *innermost* ψ-admitting sphere, contrary to what one might expect from the intuitive explanation. But if the condition in (2) is satisfied for some sphere S, then it is also satisfied for all spheres S contains, and hence in particular for the innermost sphere.

Note also that the definition of sphere models does not require that there *is* an innermost ψ-admitting sphere: we may have an infinite sequence $S_1 \supsetneq S_2 \supsetneq \cdots \supsetneq \{w\}$ of ψ-admitting spheres, and hence no innermost ψ-admitting spheres. In that case, $\mathfrak{M}, w \Vdash \psi \mathrel{\Box\!\!\rightarrow} \chi$ iff $\psi \rightarrow \chi$ holds throughout the spheres S_i, S_{i+1}, $\dots$, for some i.

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Bibliography