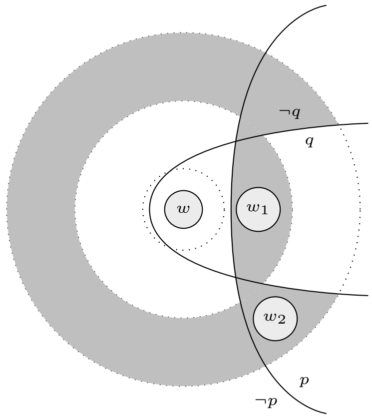

Figure 1: Counterexample to contraposition

cnt:min:cpo:
fig:contraposition

min.1 Contraposition

cnt:min:cpo:
sec

Material and strict conditionals are equivalent to their contrapositives. Counterfactuals are not. Here is an example due to Kratzer:

> If Goethe hadn't died in 1832, he would (still) be dead now.
>
> If Goethe weren't dead now, he would have died in 1832.

The first sentence is true: humans don't live hundreds of years. The second is clearly false: if Goethe weren't dead now, he would be still alive, and so couldn't have died in 1832.

cnt:min:cpo:
ex:contraposition-counterex

Example min.1. The sphere semantics invalidates contraposition, i.e., we have $p \mathrel{\Box\!\!\rightarrow} q \nvDash \neg q \mathrel{\Box\!\!\rightarrow} \neg p$. Think of p as "Goethe didn't die in 1832" and q as "Goethe is dead now." We can capture this in a model $\mathfrak{M}_1 = \langle W, O, V\rangle$ with $W = \{w, w_1, w_2\}$, $O = \{\{w\}, \{w, w_1\}, \{w, w_1, w_2\}\}$, $V(p) = \{w_1, w_2\}$ and $V(q) = \{w, w_1\}$. So w is the actual world where Goethe died in 1832 and is still dead; w_1 is the (close) world where Goethe died in, say, 1833, and is still dead; and w_2 is a (remote) world where Goethe is still alive. There is a p-admitting sphere $S = \{w, w_1\}$ and $p \rightarrow q$ is true at all worlds in it, so $\mathfrak{M}, w \Vdash p \mathrel{\Box\!\!\rightarrow} q$. However, the $\neg q$-admitting sphere $\{w, w_1, w_2\}$ contains a world, namely w_2, where q is false and p is true, so $\mathfrak{M}, w_2 \nVdash \neg q \rightarrow \neg p$.

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Bibliography