



Figure 1: Counterexample to antecedent strengthening

cnt:min:agg:  
fig:antecedent-strengthening

## min.1 Antecedent Strengthening

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sec

“Strengthening the antecedent” refers to the inference  $\varphi \rightarrow \chi \models (\varphi \wedge \psi) \rightarrow \chi$ . It is valid for the material conditional, but invalid for counterfactuals. Suppose it is true that if I were to strike this match, it would light. (That means, there is nothing wrong with the match or the matchbook surface, I will not break the match, etc.) But it is not true that if I were to light this match in outer space, it would light. So the following inference is invalid:

I the match were struck, it would light.

Therefore, if the match were struck in outer space, it would light.

The Lewis-Stalnaker account of conditionals explains this: the closest world where I light the match and I do so in outer space is much further removed from the actual world than the closest world where I light the match is. So although it’s true that the match lights in the latter, it is not in the former. And that is as it should be.

**Example min.1.** The sphere semantics invalidates the inference, i.e., we have  $p \Box \rightarrow r \not\models (p \wedge q) \Box \rightarrow r$ . Consider the model  $\mathfrak{M} = \langle W, O, V \rangle$  where  $W = \{w, w_1, w_2\}$ ,  $O_w = \{\{w\}, \{w, w_1\}, \{w, w_1, w_2\}\}$ ,  $V(p) = \{w_1, w_2\}$ ,  $V(q) = \{w_2\}$ , and  $V(r) = \{w_1\}$ . There is a  $p$ -admitting sphere  $S = \{w, w_1\}$  and  $p \rightarrow r$  is true at all worlds in it, so  $\mathfrak{M}, w \Vdash p \Box \rightarrow r$ . There is also a  $(p \wedge q)$ -admitting sphere  $S' = \{w, w_1, w_2\}$  but  $\mathfrak{M}, w_2 \not\models (p \wedge q) \rightarrow r$ , so  $\mathfrak{M}, w \not\models (p \wedge q) \Box \rightarrow r$  (see [Figure 1](#)).

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**Bibliography**