

int.1 Paradoxes of the Material Conditional

cnt:int:par:
sec One of the first to criticize the use of $\varphi \rightarrow \psi$ as a way to symbolize “if ... then ...” statements of English was C. I. Lewis. Lewis was criticizing the use of the material condition in Whitehead and Russell’s *Principia Mathematica*, who pronounced \rightarrow as “implies.” Lewis rightly complained that if \rightarrow meant “implies,” then any false proposition p implies that p implies q , since $p \rightarrow (p \rightarrow q)$ is true if p is false, and that any true proposition q implies that p implies q , since $q \rightarrow (p \rightarrow q)$ is true if q is true.

Logicians of course know that *implication*, i.e., logical entailment, is not a connective but a relation between **formulas** or statements. So we should just not read \rightarrow as “implies” to avoid confusion.¹ As long as we don’t, the particular worry that Lewis had simply does not arise: p does not “imply” q even if we think of p as standing for a false English sentence. To determine if $p \models q$ we must consider *all valuations*, and $p \not\models q$ even when we use p to symbolize a sentence which happens to be false.

But there is still something odd about “if ... then ...” statements such as Lewis’s

If the moon is made of green cheese, then $2 + 2 = 4$.

and about the inferences

The moon is not made of green cheese. Therefore, if the moon is made of green cheese, then $2 + 2 = 4$.

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Yet, if “if ... then ...” were just \rightarrow , the sentence would be unproblematically true, and the inferences unproblematically valid.

Another example of concerns the tautology $(\varphi \rightarrow \psi) \vee (\psi \rightarrow \varphi)$. This would suggest that if you take two indicative sentences S and T from the newspaper at random, the sentence “If S then T , or if T then S ” should be true.

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Bibliography

¹Reading “ \rightarrow ” as “implies” is still widely practised by mathematicians and computer scientists, although philosophers try to avoid the confusions Lewis highlighted by pronouncing it as “only if.”