

## thy.1 The Universal Partial Computable Function

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sec  
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thm:univ-comp

**Theorem thy.1.** *There is a universal partial computable function  $\text{Un}(k, x)$ . In other words, there is a function  $\text{Un}(k, x)$  such that:*

1.  $\text{Un}(k, x)$  is partial computable.
2. If  $f(x)$  is any partial computable function, then there is a natural number  $k$  such that  $f(x) \simeq \text{Un}(k, x)$  for every  $x$ .

*Proof.* Let  $\text{Un}(k, x) \simeq U(\mu s T(k, x, s))$  in Kleene's normal form theorem.  $\square$

This is just a precise way of saying that we have an effective enumeration of the partial computable functions; the idea is that if we write  $f_k$  for the function defined by  $f_k(x) = \text{Un}(k, x)$ , then the sequence  $f_0, f_1, f_2, \dots$  includes all the partial computable functions, with the property that  $f_k(x)$  can be computed "uniformly" in  $k$  and  $x$ . For simplicity, we are using a binary function that is universal for unary functions, but by coding sequences of numbers we can easily generalize this to more arguments. For example, note that if  $f(x, y, z)$  is a 3-place partial recursive function, then the function  $g(x) \simeq f((x)_0, (x)_1, (x)_2)$  is a unary recursive function. explanation

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## Bibliography