thy.1 No Universal Computable Function

cmp:thy:nou:sec

cmp:thy:nou:thm:no-univ **Theorem thy.1.** *There is no universal computable function. In other words, the universal function* $\mathrm{Un}'(k, x) = \varphi_k(x)$ *is not computable.*

Proof. This theorem says that there is no *total* computable function that is universal for the total computable functions. The proof is a simple diagonalization: if $\mathrm{Un}'(k, x)$ were total and computable, then

$$d(x) = \mathrm{Un}'(x, x) + 1$$

would also be total and computable. However, for every k, $d(k)$ is not equal to $\mathrm{Un}'(k, k)$. □

Theorem **??** above shows that we can get around this diagonalization argument, but only at the expense of allowing partial functions. It is worth trying to understand what goes wrong with the diagonalization argument, when we try to apply it in the partial case. In particular, the function $h(x) = \mathrm{Un}(x, x) + 1$ *is* partial recursive. Suppose h is the k-th function in the enumeration; what can we say about $h(k)$? explanation

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Bibliography