mac.1 Unary Representation of Numbers

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Turing machines work on sequences of symbols written on their tape. Depending on the alphabet a Turing machine uses, these sequences of symbols can represent various inputs and outputs. Of particular interest, of course, are Turing machines which compute *arithmetical* functions, i.e., functions of natural numbers. A simple way to represent positive integers is by coding them as sequences of a single symbol 1. If $n \in \mathbb{N}$, let 1^n be the empty sequence if $n = 0$, and otherwise the sequence consisting of exactly n 1's. explanation

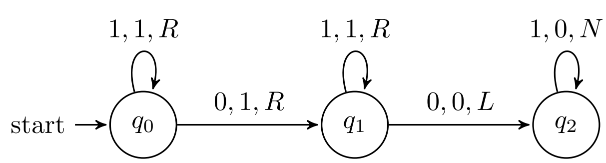
Definition mac.1 (Computation)**.** A Turing machine M *computes* the function $f\colon \mathbb{N}^n \to \mathbb{N}$ iff M halts on input

$$1^{k_1}01^{k_2}0\dots01^{k_n}$$

with output $1^{f(k_1,\dots,k_n)}$.

Example mac.2. *Addition:* Build a machine that, when given an input of two non-empty strings of 1's of length n and m, computes the function $f(n,m) = n + m$.

We want to come up with a machine that starts with two blocks of strokes on the tape and halts with one block of strokes. We first need a method to carry out. The input strokes are separated by a blank, so one method would be to write a stroke on the square containing the blank, and erase the first (or last) stroke. This would result in a block of $n+m$ 1's. Alternatively, we could proceed in a similar way to the doubler machine, by erasing a stroke from the first block, and adding one to the second block of strokes until the first block has been removed completely. We will proceed with the former example.



Problem mac.1. Trace through the configurations of the machine for input $\langle 3, 5\rangle$.

Problem mac.2. *Subtraction:* Design a Turing machine that when given an input of two non-empty strings of strokes of length n and m, where $n > m$, computes the function $f(n,m) = n - m$.

Problem mac.3. *Equality:* Design a Turing machine to compute the following function:

$$\text{equality}(x,y) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{if } x \neq y \end{cases}$$

where x and y are integers greater than 0.

Problem mac.4. Design a Turing machine to compute the function $\min(x, y)$ where x and y are positive integers represented on the tape by strings of 1's separated by a 0. You may use additional symbols in the alphabet of the machine.

The function min selects the smallest value from its arguments, so $\min(3, 5) = 3$, $\min(20, 16) = 16$, and $\min(4, 4) = 4$, and so on.

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Bibliography