Recall tracing through the configurations of the even machine earlier. The imaginary mechanism consisting of tape, read/write head, and Turing machine program is really just an intuitive way of visualizing what a Turing machine computation is. Formally, we can define the computation of a Turing machine on a given input as a sequence of configurations—and a configuration in turn is a sequence of symbols (corresponding to the contents of the tape at a given point in the computation), a number indicating the position of the read/write head, and a state. Using these, we can define what the Turing machine $M$ computes on a given input.

**Definition tur.1** (Configuration). A *configuration* of Turing machine $M = (Q, Σ, q_0, δ)$ is a triple $⟨C, n, q⟩$ where

1. $C ∈ Σ^*$ is a finite sequence of symbols from $Σ$,
2. $n ∈ N$ is a number $< \text{len}(C)$, and
3. $q ∈ Q$

Intuitively, the sequence $C$ is the content of the tape (symbols of all squares from the leftmost square to the last non-blank or previously visited square), $n$ is the number of the square the read/write head is scanning (beginning with 0 being the number of the leftmost square), and $q$ is the current state of the machine.

The potential input for a Turing machine is a sequence of symbols, usually a sequence that encodes a number in some form. The initial configuration of the Turing machine is that configuration in which we start the Turing machine to work on that input: the tape contains the tape end marker immediately followed by the input written on the squares to the right, the read/write head is scanning the leftmost square of the input (i.e., the square to the right of the left end marker), and the mechanism is in the designated start state $q_0$.

**Definition tur.2** (Initial configuration). The *initial configuration* of $M$ for input $I ∈ Σ^*$ is  

$⟨⊢ I, 1, q_0⟩$

The $⊢$ symbol is for *concatenation*—we want to ensure that there are no blanks between the left end marker and the beginning of the input.

**Definition tur.3.** We say that a configuration $⟨C, n, q⟩$ *yields* $⟨C', n', q'⟩$ *in one step* (according to $M$), iff

1. the $n$-th symbol of $C$ is $σ$,
2. the instruction set of $M$ specifies $δ(q, σ) = ⟨q', σ', D⟩$,
3. the $n$-th symbol of $C'$ is $σ'$, and
4. a) $D = L$ and $n' = n - 1$ if $n > 0$, otherwise $n' = 0$, or
    b) $D = R$ and $n' = n + 1$, or
    c) $D = N$ and $n' = n$.

5. if $n' > \text{len}(C)$, then $\text{len}(C') = \text{len}(C) + 1$ and the $n'$-th symbol of $C'$ is 0.

6. for all $i$ such that $i < \text{len}(C')$ and $i \neq n$, $C'(i) = C(i)$,

Definition tur.4. A run of $M$ on input $I$ is a sequence $C_i$ of configurations of $M$, where $C_0$ is the initial configuration of $M$ for input $I$, and each $C_i$ yields $C_{i+1}$ in one step.

We say that $M$ halts on input $I$ after $k$ steps if $C_k = \langle C, n, q \rangle$, the $n$th symbol of $C$ is $\sigma$, and $\delta(q, \sigma)$ is undefined. In that case, the output of $M$ for input $I$ is $O$, where $O$ is a string of symbols not beginning or ending in 0 such that $C = \triangleright 0^i \leadsto O \leadsto 0^j \triangleright$ for some $i, j \in \mathbb{N}$.

According to this definition, the output $O$ of $M$ always begins and ends in a symbol other than 0, or, if at time $k$ the entire tape is filled with 0 (except for the leftmost $\triangleright$), $O$ is the empty string.

Photo Credits

Bibliography