

## tur.1 Configurations and Computations

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sec

Recall tracing through the configurations of the even machine earlier. The imaginary mechanism consisting of tape, read/write head, and Turing machine program is really just in intuitive way of visualizing what a Turing machine computation is. Formally, we can define the computation of a Turing machine on a given input as a sequence of *configurations*—and a configuration in turn is a sequence of symbols (corresponding to the contents of the tape at a given point in the computation), a number indicating the position of the read/write head, and a state. Using these, we can define what the Turing machine  $M$  computes on a given input.

explanation

**Definition tur.1** (Configuration). A *configuration* of Turing machine  $M = \langle Q, \Sigma, q_0, \delta \rangle$  is a triple  $\langle C, n, q \rangle$  where

1.  $C \in \Sigma^*$  is a finite sequence of symbols from  $\Sigma$ ,
2.  $n \in \mathbb{N}$  is a number  $< \text{len}(C)$ , and
3.  $q \in Q$

Intuitively, the sequence  $C$  is the content of the tape (symbols of all squares from the leftmost square to the last non-blank or previously visited square),  $n$  is the number of the square the read/write head is scanning (beginning with 0 being the number of the leftmost square), and  $q$  is the current state of the machine.

The potential input for a Turing machine is a sequence of symbols, usually a sequence that encodes a number in some form. The initial configuration of the Turing machine is that configuration in which we start the Turing machine to work on that input: the tape contains the tape end marker immediately followed by the input written on the squares to the right, the read/write head is scanning the leftmost square of the input (i.e., the square to the right of the left end marker), and the mechanism is in the designated start state  $q_0$ .

explanation

**Definition tur.2** (Initial configuration). The *initial configuration* of  $M$  for input  $I \in \Sigma^*$  is

$$\langle \triangleright \frown I, 1, q_0 \rangle$$

The  $\frown$  symbol is for *concatenation*—we want to ensure that there are no blanks between the left end marker and the beginning of the input.

explanation

**Definition tur.3.** We say that a configuration  $\langle C, n, q \rangle$  *yields*  $\langle C', n', q' \rangle$  in *one step* (according to  $M$ ), iff

1. the  $n$ -th symbol of  $C$  is  $\sigma$ ,
2. the instruction set of  $M$  specifies  $\delta(q, \sigma) = \langle q', \sigma', D \rangle$ ,
3. the  $n$ -th symbol of  $C'$  is  $\sigma'$ , and

4. a)  $D = L$  and  $n' = n - 1$  if  $n > 0$ , otherwise  $n' = 0$ , or
  - b)  $D = R$  and  $n' = n + 1$ , or
  - c)  $D = N$  and  $n' = n$ ,
5. if  $n' > \text{len}(C)$ , then  $\text{len}(C') = \text{len}(C) + 1$  and the  $n'$ -th symbol of  $C'$  is 0.
6. for all  $i$  such that  $i < \text{len}(C')$  and  $i \neq n$ ,  $C'(i) = C(i)$ ,

**Definition tur.4.** A run of  $M$  on input  $I$  is a sequence  $C_i$  of configurations of  $M$ , where  $C_0$  is the initial configuration of  $M$  for input  $I$ , and each  $C_i$  yields  $C_{i+1}$  in one step.

We say that  $M$  halts on input  $I$  after  $k$  steps if  $C_k = \langle C, n, q \rangle$ , the  $n$ th symbol of  $C$  is  $\sigma$ , and  $\delta(q, \sigma)$  is undefined. In that case, the output of  $M$  for input  $I$  is  $O$ , where  $O$  is a string of symbols not beginning or ending in 0 such that  $C = \triangleright \frown 0^i \frown O \frown 0^j$  for some  $i, j \in \mathbb{N}$ .

explanation

According to this definition, the output  $O$  of  $M$  always begins and ends in a symbol other than 0, or, if at time  $k$  the entire tape is filled with 0 (except for the leftmost  $\triangleright$ ),  $O$  is the empty string.

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## Bibliography