

## siz.1 The Notion of Size, and Schröder-Bernstein

sfr:siz:sb:sec Here is an intuitive thought: if  $A$  is no larger than  $B$  and  $B$  is no larger than  $A$ , then  $A$  and  $B$  are equinumerous. To be honest, if this thought were *wrong*, then we could scarcely justify the thought that our defined notion of equinumerosity has anything to do with comparisons of “sizes” between sets! Fortunately, though, the intuitive thought is correct. This is justified by the Schröder-Bernstein Theorem. explanation

sfr:siz:sb:thm:schroder-bernstein **Theorem siz.1 (Schröder-Bernstein).** *If  $A \preceq B$  and  $B \preceq A$ , then  $A \approx B$ .*

In other words, if there is an injection from  $A$  to  $B$ , and an injection from  $B$  to  $A$ , then there is a bijection from  $A$  to  $B$ . explanation

This result, however, is really rather *difficult* to prove. Indeed, although Cantor stated the result, others proved it.<sup>1</sup> For now, you can (and must) take it on trust.

Fortunately, Schröder-Bernstein is *correct*, and it vindicates our thinking of the relations we defined, i.e.,  $A \approx B$  and  $A \preceq B$ , as having something to do with “size”. Moreover, Schröder-Bernstein is very *useful*. It can be difficult to think of a bijection between two equinumerous sets. The Schröder-Bernstein Theorem allows us to break the comparison down into cases so we only have to think of an injection from the first to the second, and vice-versa.

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## Bibliography

Potter, Michael. 2004. *Set Theory and its Philosophy*. Oxford: Oxford University Press.

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<sup>1</sup>For more on the history, see e.g., Potter (2004, pp. 165–6).