

Figure 1: The union  $X \cup Y$  of two sets is set of **elements** of  $X$  together with those of  $Y$ .

## set.1 Unions and Intersections

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We can define new sets by abstraction, and the property used to define the new set can mention sets we've already defined. So for instance, if  $X$  and  $Y$  are sets, the set  $\{x : x \in X \vee x \in Y\}$  defines a set which consists of all those objects which are **elements** of either  $X$  or  $Y$ , i.e., it's the set that combines the **elements** of  $X$  and  $Y$ . This operation on sets—combining them—is very useful and common, and so we give it a name and a symbol.

explanation

**Definition set.1** (Union). The *union* of two sets  $X$  and  $Y$ , written  $X \cup Y$ , is the set of all things which are **elements** of  $X$ ,  $Y$ , or both.

$$X \cup Y = \{x : x \in X \vee x \in Y\}$$

**Example set.2.** Since the multiplicity of **elements** doesn't matter, the union of two sets which have **an element** in common contains that **element** only once, e.g.,  $\{a, b, c\} \cup \{a, 0, 1\} = \{a, b, c, 0, 1\}$ .

The union of a set and one of its subsets is just the bigger set:  $\{a, b, c\} \cup \{a\} = \{a, b, c\}$ .

The union of a set with the empty set is identical to the set:  $\{a, b, c\} \cup \emptyset = \{a, b, c\}$ .

**Problem set.1.** Prove rigorously that if  $X \subseteq Y$ , then  $X \cup Y = Y$ .

The operation that forms the set of all **elements** that  $X$  and  $Y$  have in common is called their *intersection*.

**Definition set.3** (Intersection). The *intersection* of two sets  $X$  and  $Y$ , written  $X \cap Y$ , is the set of all things which are **elements** of both  $X$  and  $Y$ .

$$X \cap Y = \{x : x \in X \wedge x \in Y\}$$

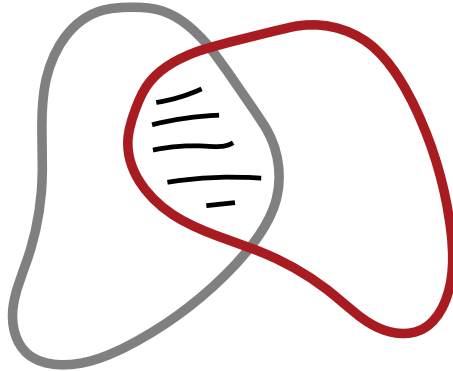


Figure 2: The intersection  $X \cap Y$  of two sets is the set of **elements** they have in common.

Two sets are called *disjoint* if their intersection is empty. This means they have no **elements** in common.

**Example set.4.** If two sets have no **elements** in common, their intersection is empty:  $\{a, b, c\} \cap \{0, 1\} = \emptyset$ .

If two sets do have **elements** in common, their intersection is the set of all those:  $\{a, b, c\} \cap \{a, b, d\} = \{a, b\}$ .

The intersection of a set with one of its subsets is just the smaller set:  $\{a, b, c\} \cap \{a, b\} = \{a, b\}$ .

The intersection of any set with the empty set is empty:  $\{a, b, c\} \cap \emptyset = \emptyset$ .

**Problem set.2.** Prove rigorously that if  $X \subseteq Y$ , then  $X \cap Y = X$ .

explanation

We can also form the union or intersection of more than two sets. An elegant way of dealing with this in general is the following: suppose you collect all the sets you want to form the union (or intersection) of into a single set. Then we can define the union of all our original sets as the set of all objects which belong to at least one **element** of the set, and the intersection as the set of all objects which belong to every **element** of the set.

**Definition set.5.** If  $Z$  is a set of sets, then  $\bigcup Z$  is the set of **elements** of **elements** of  $Z$ :

$$\bigcup Z = \{x : x \text{ belongs to an element of } Z\}, \text{ i.e.,}$$

$$\bigcup Z = \{x : \text{there is a } Y \in Z \text{ so that } x \in Y\}$$

**Definition set.6.** If  $Z$  is a set of sets, then  $\bigcap Z$  is the set of objects which all elements of  $Z$  have in common:

$$\bigcap Z = \{x : x \text{ belongs to every element of } Z\}, \text{ i.e.,}$$

$$\bigcap Z = \{x : \text{for all } Y \in Z, x \in Y\}$$

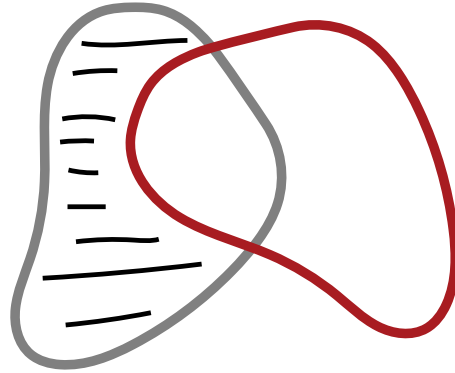


Figure 3: The difference  $X \setminus Y$  of two sets is the set of those **elements** of  $X$  which are not also elements of  $Y$ .

**Example set.7.** Suppose  $Z = \{\{a, b\}, \{a, d, e\}, \{a, d\}\}$ . Then  $\bigcup Z = \{a, b, d, e\}$  and  $\bigcap Z = \{a\}$ .

We could also do the same for a sequence of sets  $X_1, X_2, \dots$

$$\bigcup_i X_i = \{x : x \text{ belongs to one of the } X_i\}$$
$$\bigcap_i X_i = \{x : x \text{ belongs to every } X_i\}.$$

**Definition set.8** (Difference). The *difference*  $X \setminus Y$  is the set of all **elements** of  $X$  which are not also **elements** of  $Y$ , i.e.,

$$X \setminus Y = \{x : x \in X \text{ and } x \notin Y\}.$$

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## Bibliography