

## set.1 Some Important Sets

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**Example set.1.** We will mostly be dealing with sets whose **elements** are mathematical objects. Four such sets are important enough to have specific names:

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

the set of natural numbers

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$$

the set of integers

$$\mathbb{Q} = \{m/n : m, n \in \mathbb{Z} \text{ and } n \neq 0\}$$

the set of rationals

$$\mathbb{R} = (-\infty, \infty)$$

the set of real numbers (the continuum)

These are all *infinite* sets, that is, they each have infinitely many **elements**.

As we move through these sets, we are adding *more* numbers to our stock. Indeed, it should be clear that  $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$ : after all, every natural number is an integer; every integer is a rational; and every rational is a real. Equally, it should be clear that  $\mathbb{N} \subsetneq \mathbb{Z} \subsetneq \mathbb{Q}$ , since  $-1$  is an integer but not a natural number, and  $1/2$  is rational but not integer. It is less obvious that  $\mathbb{Q} \subsetneq \mathbb{R}$ , i.e., that there are some real numbers which are not rational.

We'll sometimes also use the set of positive integers  $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$  and the set containing just the first two natural numbers  $\mathbb{B} = \{0, 1\}$ .

**Example set.2 (Strings).** Another interesting example is the set  $A^*$  of *finite strings* over an alphabet  $A$ : any finite sequence of elements of  $A$  is a string over  $A$ . We include the *empty string*  $\Lambda$  among the strings over  $A$ , for every alphabet  $A$ . For instance,

$$\mathbb{B}^* = \{\Lambda, 0, 1, 00, 01, 10, 11, \\ 000, 001, 010, 011, 100, 101, 110, 111, 0000, \dots\}.$$

If  $x = x_1 \dots x_n \in A^*$  is a string consisting of  $n$  “letters” from  $A$ , then we say *length* of the string is  $n$  and write  $\text{len}(x) = n$ .

**Example set.3 (Infinite sequences).** For any set  $A$  we may also consider the set  $A^\omega$  of infinite sequences of **elements** of  $A$ . An infinite sequence  $a_1 a_2 a_3 a_4 \dots$  consists of a one-way infinite list of objects, each one of which is **an element** of  $A$ .

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**Bibliography**