set.1  Some Important Sets

Example set.1. We will mostly be dealing with sets whose elements are mathematical objects. Four such sets are important enough to have specific names:

\[\mathbb{N} = \{0, 1, 2, 3, \ldots\}\]  
the set of natural numbers

\[\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}\]  
the set of integers

\[\mathbb{Q} = \left\{ \frac{m}{n} : m, n \in \mathbb{Z} \text{ and } n \neq 0 \right\}\]  
the set of rationals

\[\mathbb{R} = (-\infty, \infty)\]  
the set of real numbers (the continuum)

These are all infinite sets, that is, they each have infinitely many elements.

As we move through these sets, we are adding more numbers to our stock. Indeed, it should be clear that \(\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}\): after all, every natural number is an integer; every integer is a rational; and every rational is a real. Equally, it should be clear that \(\mathbb{N} \subsetneq \mathbb{Z} \subsetneq \mathbb{Q}\), since \(-1\) is an integer but not a natural number, and \(\frac{1}{2}\) is rational but not integer. It is less obvious that \(\mathbb{R} \subsetneq \mathbb{Q}\), i.e., that there are some real numbers which are not rational.

We’ll sometimes also use the set of positive integers \(\mathbb{Z}^+ = \{1, 2, 3, \ldots\}\) and the set containing just the first two natural numbers \(\mathbb{B} = \{0, 1\}\).

Example set.2 (Strings). Another interesting example is the set \(A^*\) of finite strings over an alphabet \(A\): any finite sequence of elements of \(A\) is a string over \(A\). We include the empty string \(\Lambda\) among the strings over \(A\), for every alphabet \(A\). For instance,

\[A^* = \{A, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111, 0000, \ldots\}\].

If \(x = x_1 \ldots x_n \in A^*\) is a string consisting of \(n\) “letters” from \(A\), then we say length of the string is \(n\) and write \(\text{len}(x) = n\).

Example set.3 (Infinite sequences). For any set \(A\) we may also consider the set \(A^\omega\) of infinite sequences of elements of \(A\). An infinite sequence \(a_1 a_2 a_3 a_4 \ldots\) consists of a one-way infinite list of objects, each one of which is an element of \(A\).