

## set.1 Some Important Sets

sfr:set:set:  
sec

**Example set.1.** Mostly we'll be dealing with sets that have mathematical objects as members. You will remember the various sets of numbers:  $\mathbb{N}$  is the set of *natural* numbers  $\{0, 1, 2, 3, \dots\}$ ;  $\mathbb{Z}$  the set of *integers*,

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\};$$

$\mathbb{Q}$  the set of *rational* numbers ( $\mathbb{Q} = \{z/n : z \in \mathbb{Z}, n \in \mathbb{N}, n \neq 0\}$ ); and  $\mathbb{R}$  the set of *real* numbers. These are all *infinite* sets, that is, they each have infinitely many **elements**. As it turns out,  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$  have the same number of **elements**, while  $\mathbb{R}$  has a whole bunch more— $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$  are “**enumerable** and infinite” whereas  $\mathbb{R}$  is “**non-enumerable**”.

We'll sometimes also use the set of positive integers  $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$  and the set containing just the first two natural numbers  $\mathbb{B} = \{0, 1\}$ .

**Example set.2** (Strings). Another interesting example is the set  $A^*$  of *finite strings* over an alphabet  $A$ : any finite sequence of elements of  $A$  is a string over  $A$ . We include the *empty string*  $\Lambda$  among the strings over  $A$ , for every alphabet  $A$ . For instance,

$$\mathbb{B}^* = \{\Lambda, 0, 1, 00, 01, 10, 11, \\ 000, 001, 010, 011, 100, 101, 110, 111, 0000, \dots\}.$$

If  $x = x_1 \dots x_n \in A^*$  is a string consisting of  $n$  “letters” from  $A$ , then we say *length* of the string is  $n$  and write  $\text{len}(x) = n$ .

**Example set.3** (Infinite sequences). For any set  $A$  we may also consider the set  $A^\omega$  of infinite sequences of **elements** of  $A$ . An infinite sequence  $a_1 a_2 a_3 a_4 \dots$  consists of a one-way infinite list of objects, each one of which is **an element** of  $A$ .

## Photo Credits

## Bibliography