

set.1 Some Important Sets

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Example set.1. Mostly we'll be dealing with sets that have mathematical objects as members. You will remember the various sets of numbers: \mathbb{N} is the set of *natural* numbers $\{0, 1, 2, 3, \dots\}$; \mathbb{Z} the set of *integers*,

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\};$$

\mathbb{Q} the set of *rational* numbers ($\mathbb{Q} = \{z/n : z \in \mathbb{Z}, n \in \mathbb{N}, n \neq 0\}$); and \mathbb{R} the set of *real* numbers. These are all *infinite* sets, that is, they each have infinitely many **elements**. As it turns out, \mathbb{N} , \mathbb{Z} , \mathbb{Q} have the same number of **elements**, while \mathbb{R} has a whole bunch more— \mathbb{N} , \mathbb{Z} , \mathbb{Q} are “**enumerable** and infinite” whereas \mathbb{R} is “**non-enumerable**”.

We'll sometimes also use the set of positive integers $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$ and the set containing just the first two natural numbers $\mathbb{B} = \{0, 1\}$.

Example set.2 (Strings). Another interesting example is the set A^* of *finite strings* over an alphabet A : any finite sequence of elements of A is a string over A . We include the *empty string* Λ among the strings over A , for every alphabet A . For instance,

$$\mathbb{B}^* = \{\Lambda, 0, 1, 00, 01, 10, 11, \\ 000, 001, 010, 011, 100, 101, 110, 111, 0000, \dots\}.$$

If $x = x_1 \dots x_n \in A^*$ is a string consisting of n “letters” from A , then we say *length* of the string is n and write $\text{len}(x) = n$.

Example set.3 (Infinite sequences). For any set A we may also consider the set A^ω of infinite sequences of **elements** of A . An infinite sequence $a_1 a_2 a_3 a_4 \dots$ consists of a one-way infinite list of objects, each one of which is **an element** of A .

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Bibliography