set.1 Basics

sfr:set:bas:sec Sets are the most fundamental building blocks of mathematical objects. In fact, almost every mathematical object can be seen as a set of some kind. In logic, as in other parts of mathematics, sets and set-theoretical talk is ubiquitous. So it will be important to discuss what sets are, and introduce the notations necessary to talk about sets and operations on sets in a standard way. explanation

Definition set.1 (Set)**.** A *set* is a collection of objects, considered independently of the way it is specified, of the order of the objects in the set, or of their multiplicity. The objects making up the set are called *elements* or *members* of the set. If a is an element of a set X, we write $a \in X$ (otherwise, $a \notin X$). The set which has no elements is called the *empty* set and denoted by the symbol $\emptyset$.

Example set.2. Whenever you have a bunch of objects, you can collect them together in a set. The set of Richard's siblings, for instance, is a set that contains one person, and we could write it as $S = \{\text{Ruth}\}$. In general, when we have some objects a_1, …, a_n, then the set consisting of exactly those objects is written $\{a_1, \dots, a_n\}$. Frequently we'll specify a set by some property that its elements share—as we just did, for instance, by specifying S as the set of Richard's siblings. We'll use the following shorthand notation for that: $\{x : \dots x \dots\}$, where the $\dots x \dots$ stands for the property that x has to have in order to be counted among the elements of the set. In our example, we could have specified S also as

$$S = \{x : x \text{ is a sibling of Richard}\}.$$

When we say that sets are independent of the way they are specified, we mean that the elements of a set are all that matters. For instance, it so happens that explanation

$$\begin{aligned}
&\{\text{Nicole}, \text{Jacob}\},\\
&\{x : \text{is a niece or nephew of Richard}\}, \text{ and}\\
&\{x : \text{is a child of Ruth}\}
\end{aligned}$$

are three ways of specifying one and the same set.

Saying that sets are considered independently of the order of their elements and their multiplicity is a fancy way of saying that

$$\begin{aligned}
&\{\text{Nicole}, \text{Jacob}\} \text{ and}\\
&\{\text{Jacob}, \text{Nicole}\}
\end{aligned}$$

are two ways of specifying the same set; and that

$$\begin{aligned}
&\{\text{Nicole}, \text{Jacob}\} \text{ and}\\
&\{\text{Jacob}, \text{Nicole}, \text{Nicole}\}
\end{aligned}$$

are also two ways of specifying the same set. In other words, all that matters is which elements a set has. The elements of a set are not ordered and each element occurs only once. When we *specify* or *describe* a set, elements may occur multiple times and in different orders, but any descriptions that only differ in the order of elements or in how many times elements are listed describes the same set.

Definition set.3 (Extensionality)**.** If X and Y are sets, then X and Y are *identical*, $X = Y$, iff every element of X is also an element of Y, and vice versa.

explanation

Extensionality gives us a way for showing that sets are identical: to show that $X = Y$, show that whenever $x \in X$ then also $x \in Y$, and whenever $y \in Y$ then also $y \in X$.

Problem set.1. Show that there is only one empty set, i.e., show that if X and Y are sets without members, then $X = Y$.

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Bibliography