

fun.1 Partial Functions

sfr:fun:par:sec It is sometimes useful to relax the definition of function so that it is not required explanation that the output of the function is defined for all possible inputs. Such mappings are called *partial functions*.

Definition fun.1. A *partial function* $f: X \rightarrow Y$ is a mapping which assigns to every **element** of X at most one **element** of Y . If f assigns an element of Y to $x \in X$, we say $f(x)$ is *defined*, and otherwise *undefined*. If $f(x)$ is defined, we write $f(x) \downarrow$, otherwise $f(x) \uparrow$. The *domain* of a partial function f is the subset of X where it is defined, i.e., $\text{dom}(f) = \{x : f(x) \downarrow\}$.

Example fun.2. Every function $f: X \rightarrow Y$ is also a partial function. Partial functions that are defined everywhere on X —i.e., what we so far have simply called a function—are also called *total* functions.

Example fun.3. The partial function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 1/x$ is undefined for $x = 0$, and defined everywhere else.

Problem fun.1. Given $f: X \rightarrow Y$, define the partial function $g: Y \rightarrow X$ by: for any $y \in Y$, if there is a unique $x \in X$ such that $f(x) = y$, then $g(y) = x$; otherwise $g(y) \uparrow$. Show that if f is injective, then $g(f(x)) = x$ for all $x \in \text{dom}(f)$, and $f(g(y)) = y$ for all $y \in \text{ran}(f)$.

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Bibliography