fun.1 Isomorphism

sfr:fun:iso:sec

An *isomorphism* is a bijection that preserves the structure of the sets it relates, where structure is a matter of the relationships that obtain between the elements of the sets. Consider the following two sets $X = \{1, 2, 3\}$ and $Y = \{4, 5, 6\}$. These sets are both structured by the relations successor, less than, and greater than. An isomorphism between the two sets is a bijection that preserves those structures. So a bijective function $f\colon X \to Y$ is an isomorphism if, $i < j$ iff $f(i) < f(j)$, $i > j$ iff $f(i) > f(j)$, and j is the successor of i iff $f(j)$ is the successor of $f(i)$.

explanation

Definition fun.1 (Isomorphism)**.** Let U be the pair $\langle X, R\rangle$ and V be the pair $\langle Y, S\rangle$ such that X and Y are sets and R and S are relations on X and Y respectively. A bijection f from X to Y is an *isomorphism* from U to V iff it preserves the relational structure, that is, for any x_1 and x_2 in X, $\langle x_1, x_2\rangle \in R$ iff $\langle f(x_1), f(x_2)\rangle \in S$.

Example fun.2. Consider the following two sets $X = \{1, 2, 3\}$ and $Y = \{4, 5, 6\}$, and the relations less than and greater than. The function $f\colon X \to Y$ where $f(x) = 7 - x$ is an isomorphism between $\langle X, <\rangle$ and $\langle Y, >\rangle$.

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Bibliography