

fun.1 Functions and Relations

sfr:fun:rel:
sec A function which maps **elements** of X to **elements** of Y obviously defines explanation a relation between X and Y , namely the relation which holds between x and y iff $f(x) = y$. In fact, we might even—if we are interested in reducing the building blocks of mathematics for instance—*identify* the function f with this relation, i.e., with a set of pairs. This then raises the question: which relations define functions in this way?

Definition fun.1 (Graph of a function). Let $f: X \rightarrow Y$ be a partial function. The *graph* of f is the relation $R_f \subseteq X \times Y$ defined by

$$R_f = \{(x, y) : f(x) = y\}.$$

Proposition fun.2. *Suppose $R \subseteq X \times Y$ has the property that whenever Rxy and Rxy' then $y = y'$. Then R is the graph of the partial function $f: X \rightarrow Y$ defined by: if there is a y such that Rxy , then $f(x) = y$, otherwise $f(x) \uparrow$. If R is also serial, i.e., for each $x \in X$ there is a $y \in Y$ such that Rxy , then f is total.*

Proof. Suppose there is a y such that Rxy . If there were another $y' \neq y$ such that Rxy' , the condition on R would be violated. Hence, if there is a y such that Rxy , that y is unique, and so f is well-defined. Obviously, $R_f = R$ and f is total if R is serial. \square

Problem fun.1. Suppose $f: X \rightarrow Y$ and $g: Y \rightarrow Z$. Show that the graph of $(g \circ f)$ is $R_f \mid R_g$.

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Bibliography