

Figure 1: A **surjective** function has every **element** of the codomain as a value.



Figure 2: An **injective** function never maps two different arguments to the same value.

fun.1 Kinds of Functions

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Definition fun.1 (**Surjective** function). A function $f: X \rightarrow Y$ is **surjective** iff Y is also the range of f , i.e., for every $y \in Y$ there is at least one $x \in X$ such that $f(x) = y$.

If you want to show that a function is **surjective**, then you need to show that every object in the codomain is the output of the function given some input or other. explanation

Definition fun.2 (**Injective** function). A function $f: X \rightarrow Y$ is **injective** iff for each $y \in Y$ there is at most one $x \in X$ such that $f(x) = y$.

Any function pairs each possible input with a unique output. An **injective** function has a unique input for each possible output. If you want to show that a function f is **injective**, you need to show that for any **elements** x and x' of the domain, if $f(x) = f(x')$, then $x = x'$. explanation

An example of a function which is neither **injective**, nor **surjective**, is the constant function $f: \mathbb{N} \rightarrow \mathbb{N}$ where $f(x) = 1$.

An example of a function which is both **injective** and **surjective** is the identity function $f: \mathbb{N} \rightarrow \mathbb{N}$ where $f(x) = x$.

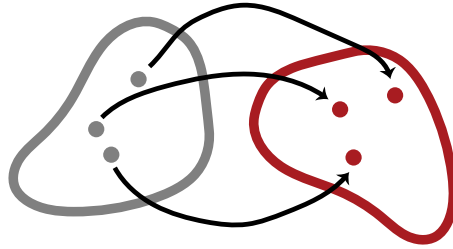


Figure 3: A **bijective** function uniquely pairs the elements of the codomain with those of the domain.

The successor function $f: \mathbb{N} \rightarrow \mathbb{N}$ where $f(x) = x + 1$ is **injective**, but not **surjective**.

The function

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ \frac{x+1}{2} & \text{if } x \text{ is odd.} \end{cases}$$

is **surjective**, but not **injective**.

Definition fun.3 (Bijection). A function $f: X \rightarrow Y$ is **bijective** iff it is both **surjective** and **injective**. We call such a function a **bijection** from X to Y (or between X and Y).

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Bibliography