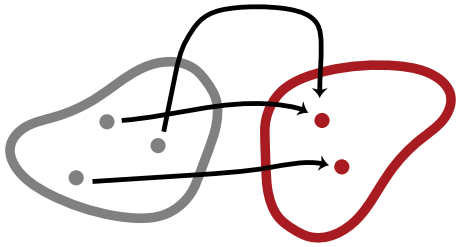

Figure 1: A surjective function has every element of the codomain as a value.

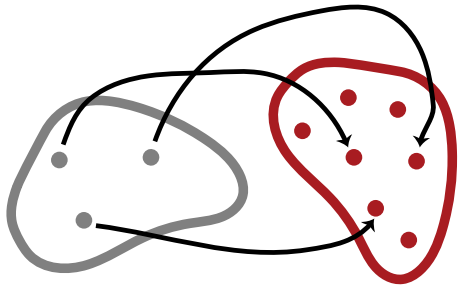

Figure 2: An injective function never maps two different arguments to the same value.

fun.1 Kinds of Functions

sfr:fun:kin:sec

Definition fun.1 (Surjective function)**.** A function $f\colon X \to Y$ is *surjective* iff Y is also the range of f, i.e., for every $y \in Y$ there is at least one $x \in X$ such that $f(x) = y$.

If you want to show that a function is surjective, then you need to show that every object in the codomain is the output of the function given some input or other. explanation

Definition fun.2 (Injective function)**.** A function $f\colon X \to Y$ is *injective* iff for each $y \in Y$ there is at most one $x \in X$ such that $f(x) = y$.

Any function pairs each possible input with a unique output. An injective function has a unique input for each possible output. If you want to show that a function f is injective, you need to show that for any elements x and x' of the domain, if $f(x) = f(x')$, then $x = x'$. explanation

An example of a function which is neither injective, nor surjective, is the constant function $f\colon \mathbb{N} \to \mathbb{N}$ where $f(x) = 1$.

An example of a function which is both injective and surjective is the identity function $f\colon \mathbb{N} \to \mathbb{N}$ where $f(x) = x$.

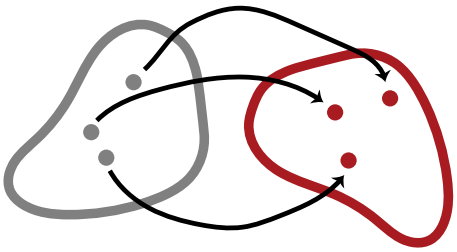

Figure 3: A bijective function uniquely pairs the elements of the codomain with those of the domain.

The successor function $f\colon \mathbb{N} \to \mathbb{N}$ where $f(x) = x + 1$ is injective, but not surjective.

The function

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ \frac{x+1}{2} & \text{if } x \text{ is odd.} \end{cases}$$

is surjective, but not injective.

Definition fun.3 (Bijection)**.** A function $f\colon X \to Y$ is *bijective* iff it is both surjective and injective. We call such a function a *bijection* from X to Y (or between X and Y).

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Bibliography