fun.1 Kinds of Functions

Definition fun.1 (Surjective function). A function \( f : X \rightarrow Y \) is surjective iff \( Y \) is also the range of \( f \), i.e., for every \( y \in Y \) there is at least one \( x \in X \) such that \( f(x) = y \).

If you want to show that a function is surjective, then you need to show that every object in the codomain is the output of the function given some input or other.

Definition fun.2 (Injective function). A function \( f : X \rightarrow Y \) is injective iff for each \( y \in Y \) there is at most one \( x \in X \) such that \( f(x) = y \).

Any function pairs each possible input with a unique output. An injective function has a unique input for each possible output. If you want to show that a function \( f \) is injective, you need to show that for any elements \( x \) and \( x' \) of the domain, if \( f(x) = f(x') \), then \( x = x' \).

An example of a function which is neither injective, nor surjective, is the constant function \( f : \mathbb{N} \rightarrow \mathbb{N} \) where \( f(x) = 1 \).

An example of a function which is both injective and surjective is the identity function \( f : \mathbb{N} \rightarrow \mathbb{N} \) where \( f(x) = x \).
The successor function $f : \mathbb{N} \to \mathbb{N}$ where $f(x) = x + 1$ is injective, but not surjective.

The function

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \ \text{is even} \\ \frac{x+1}{2} & \text{if } x \ \text{is odd} \end{cases}$$

is surjective, but not injective.

**Definition** fun.3 (Bijection). A function $f : X \to Y$ is **bijective** iff it is both surjective and injective. We call such a function a **bijection** from $X$ to $Y$ (or between $X$ and $Y$).

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Bibliography