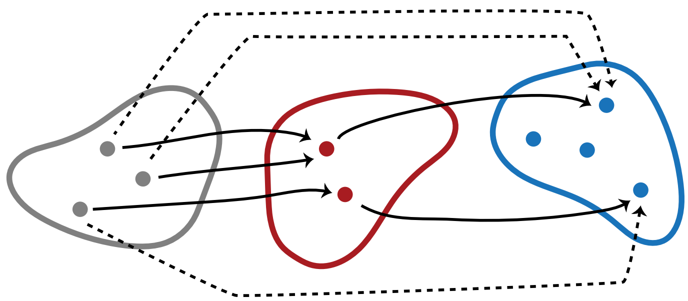

Figure 1: The composition $g \circ f$ of two functions f and g.

fun.1 Composition of Functions

sfr:fun:cmp:sec We have already seen that the inverse f^{-1} of a bijective function f is itself a function. It is also possible to compose functions f and g to define a new function by first applying f and then g. Of course, this is only possible if the ranges and domains match, i.e., the range of f must be a subset of the domain of g. explanation

Definition fun.1 (Composition)**.** Let $f\colon X \to Y$ and $g\colon Y \to Z$. The *composition* of f with g is the function $(g \circ f)\colon X \to Z$, where $(g \circ f)(x) = g(f(x))$.

The function $(g \circ f)\colon X \to Z$ pairs each member of X with a member of Z. We specify which member of Z a member of X is paired with as follows—given an input $x \in X$, first apply the function f to x, which will output some $y \in Y$. Then apply the function g to y, which will output some $z \in Z$. explanation

Example fun.2. Consider the functions $f(x) = x + 1$, and $g(x) = 2x$. What function do you get when you compose these two? $(g \circ f)(x) = g(f(x))$. So that means for every natural number you give this function, you first add one, and then you multiply the result by two. So their composition is $(g \circ f)(x) = 2(x+1)$.

Problem fun.1. Show that if $f\colon X \to Y$ and $g\colon Y \to Z$ are both injective, then $g \circ f\colon X \to Z$ is injective.

Problem fun.2. Show that if $f\colon X \to Y$ and $g\colon Y \to Z$ are both surjective, then $g \circ f\colon X \to Z$ is surjective.

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Bibliography