



Figure 1: The composition  $g \circ f$  of two functions  $f$  and  $g$ .

## fun.1 Composition of Functions

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We have already seen that the inverse  $f^{-1}$  of a **bijjective** function  $f$  is itself a function. It is also possible to compose functions  $f$  and  $g$  to define a new function by first applying  $f$  and then  $g$ . Of course, this is only possible if the ranges and domains match, i.e., the range of  $f$  must be a subset of the domain of  $g$ .

explanation

**Definition fun.1** (Composition). Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$ . The *composition* of  $f$  with  $g$  is the function  $(g \circ f): X \rightarrow Z$ , where  $(g \circ f)(x) = g(f(x))$ .

The function  $(g \circ f): X \rightarrow Z$  pairs each member of  $X$  with a member of  $Z$ . We specify which member of  $Z$  a member of  $X$  is paired with as follows—given an input  $x \in X$ , first apply the function  $f$  to  $x$ , which will output some  $y \in Y$ . Then apply the function  $g$  to  $y$ , which will output some  $z \in Z$ .

explanation

**Example fun.2.** Consider the functions  $f(x) = x + 1$ , and  $g(x) = 2x$ . What function do you get when you compose these two?  $(g \circ f)(x) = g(f(x))$ . So that means for every natural number you give this function, you first add one, and then you multiply the result by two. So their composition is  $(g \circ f)(x) = 2(x+1)$ .

**Problem fun.1.** Show that if  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  are both **injective**, then  $g \circ f: X \rightarrow Z$  is **injective**.

**Problem fun.2.** Show that if  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  are both **surjective**, then  $g \circ f: X \rightarrow Z$  is **surjective**.

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## Bibliography