



Figure 1: The composition $g \circ f$ of two functions f and g .

fun.1 Composition of Functions

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We have already seen that the inverse f^{-1} of a **bijjective** function f is itself a function. It is also possible to compose functions f and g to define a new function by first applying f and then g . Of course, this is only possible if the ranges and domains match, i.e., the range of f must be a subset of the domain of g .

explanation

Definition fun.1 (Composition). Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$. The *composition* of f with g is the function $(g \circ f): X \rightarrow Z$, where $(g \circ f)(x) = g(f(x))$.

The function $(g \circ f): X \rightarrow Z$ pairs each member of X with a member of Z . We specify which member of Z a member of X is paired with as follows—given an input $x \in X$, first apply the function f to x , which will output some $y \in Y$. Then apply the function g to y , which will output some $z \in Z$.

explanation

Example fun.2. Consider the functions $f(x) = x + 1$, and $g(x) = 2x$. What function do you get when you compose these two? $(g \circ f)(x) = g(f(x))$. So that means for every natural number you give this function, you first add one, and then you multiply the result by two. So their composition is $(g \circ f)(x) = 2(x+1)$.

Problem fun.1. Show that if $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are both **injective**, then $g \circ f: X \rightarrow Z$ is **injective**.

Problem fun.2. Show that if $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are both **surjective**, then $g \circ f: X \rightarrow Z$ is **surjective**.

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Bibliography