fun.1 Composition of Functions

We can define a new function by composing two functions, \( f \) and \( g \), i.e., by first applying \( f \) and then \( g \). Of course, this is only possible if the ranges and domains match, i.e., the range of \( f \) must be a subset of the domain of \( g \).

A diagram might help to explain the idea of composition. In Figure 1, we depict two functions \( f: A \to B \) and \( g: B \to C \) and their composition \((g \circ f)\). The function \((g \circ f): A \to C\) pairs each element of \( A \) with an element of \( C \). We specify which element of \( C \) an element of \( A \) is paired with as follows: given an input \( x \in A \), first apply the function \( f \) to \( x \), which will output some \( f(x) = y \in B \), then apply the function \( g \) to \( y \), which will output some \( g(f(x)) = g(y) = z \in C \).

Definition fun.1 (Composition). Let \( f: A \to B \) and \( g: B \to C \) be functions. The composition of \( f \) with \( g \) is \( g \circ f: A \to C \), where \((g \circ f)(x) = g(f(x))\).

Example fun.2. Consider the functions \( f(x) = x + 1 \), and \( g(x) = 2x \). Since \((g \circ f)(x) = g(f(x))\), for each input \( x \) you must first take its successor, then multiply the result by two. So their composition is given by \((g \circ f)(x) = 2(x+1)\).

Problem fun.1. Show that if \( f: A \to B \) and \( g: B \to C \) are both injective, then \( g \circ f: A \to C \) is injective.

Problem fun.2. Show that if \( f: A \to B \) and \( g: B \to C \) are both surjective, then \( g \circ f: A \to C \) is surjective.

Problem fun.3. Suppose \( f: A \to B \) and \( g: B \to C \). Show that the graph of \( g \circ f \) is \( R_f \mid R_g \).
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Bibliography