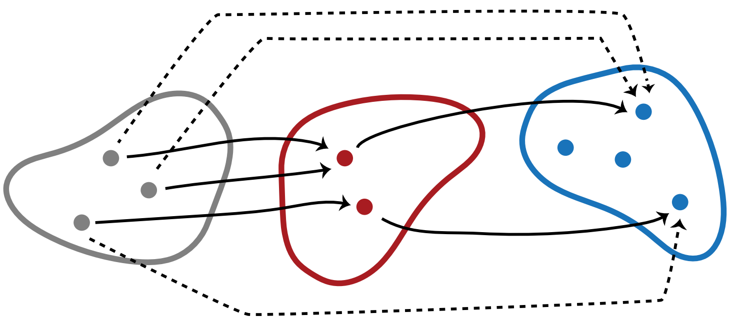

Figure 1: The composition $g \circ f$ of two functions f and g.

sfr:fun:cmp:
fig:composition

fun.1 Composition of Functions

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sec

We can define a new function by composing two functions, f and g, i.e., by first applying f and then g. Of course, this is only possible if the ranges and domains match, i.e., the range of f must be a subset of the domain of g. explanation

A diagram might help to explain the idea of composition. In Figure 1, we depict two functions $f : A \to B$ and $g : B \to C$ and their composition $(g \circ f)$. The function $(g \circ f) : A \to C$ pairs each element of A with an element of C. We specify which element of C an element of A is paired with as follows: given an input $x \in A$, first apply the function f to x, which will output some $f(x) = y \in B$, then apply the function g to y, which will output some $g(f(x)) = g(y) = z \in C$.

Definition fun.1 (Composition). Let $f : A \to B$ and $g : B \to C$ be functions. The *composition* of f with g is $g \circ f : A \to C$, where $(g \circ f)(x) = g(f(x))$.

Example fun.2. Consider the functions $f(x) = x + 1$, and $g(x) = 2x$. Since $(g \circ f)(x) = g(f(x))$, for each input x you must first take its successor, then multiply the result by two. So their composition is given by $(g \circ f)(x) = 2(x + 1)$.

Problem fun.1. Show that if $f : A \to B$ and $g : B \to C$ are both injective, then $g \circ f : A \to C$ is injective.

Problem fun.2. Show that if $f : A \to B$ and $g : B \to C$ are both surjective, then $g \circ f : A \to C$ is surjective.

Problem fun.3. Suppose $f : A \to B$ and $g : B \to C$. Show that the graph of $g \circ f$ is $R_f \mid R_g$.

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Bibliography