

z.1 Union

sth:z:union:
sec ?? gave us intersections. But if we want arbitrary unions to exist, we need to lay down another axiom:

Axiom (Union). For any set A , the set $\bigcup A = \{x : (\exists b \in A)x \in b\}$ exists.

$$\forall A \exists U \forall x (x \in U \leftrightarrow (\exists b \in A)x \in b)$$

This axiom is also justified by the cumulative-iterative conception. Let A be a set, so A is formed at some stage S (by *Stages-are-key*). Every member of A was formed *before* S (by *Stages-accumulate*); so, reasoning similarly, every member of every member of A was formed before S . Thus all of *those* sets are available before S , to be formed into a set at S . And that set is just $\bigcup A$.

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Bibliography